13.003
Computational Geometry and Visualization
—
Problem Set 1

Massachusetts Institute of Technology
Department of Ocean Engineering
Cambridge, MA 02139-4307

Out: February 7, 2000
Due: February 14, 2000

This problem set will be an introduction to computation and visualization using MATLAB. Solve the equations analytically by hand and use MATLAB for numerical analysis and plots. Use the handout distributed in class for more information on using MATLAB. If you prefer, you may use another language such as C for your computations.

Submit all plots and paper work with clear explanations of your work. E-mail your Matlab source code to gitano@mit.edu in one file (write your code in a single ‘M’ file).

1. Using your favorite web browser, please go to the 13.003 home page <http://deslab.mit.edu/DesignLab/13.003/>. We will keep this site up to date with the latest information for this course (announcements, handouts, homeworks, hints, etc.). Even if you have already done so on paper, please register for the course electronically at <http://deslab.mit.edu/DesignLab/13.003/form.html>.

2. The sheer line (edge of deck) of an 8ft yacht tender from the Haffenreffer-Herreshoff collection is represented by two cubic planar parametric curves $r_1(u) = \{x_1(u), y_1(u), z_1(u)\}$ and $r_2(v) = \{x_2(v), y_2(v), z_2(v)\}$, where $0 \leq u, v \leq 1$:

   \[
   r_1(u) = \begin{bmatrix} x_1(u) \\ y_1(u) \\ z_1(u) \end{bmatrix} = \begin{bmatrix} -105.3372u^3 + 257.0999u^2 + 232.2372u + 57.0275u^3 - 279.1296u^2 + 392.1021u + 38.7212u^3 - 19.1985u^2 - 92.5227u + 170 \\ 38.7212u^3 - 19.1985u^2 - 92.5227u + 170 \end{bmatrix} \\
   \]

   \[
   r_2(v) = \begin{bmatrix} x_2(v) \\ y_2(v) \\ z_2(v) \end{bmatrix} = \begin{bmatrix} -2.9684v^3 - 3.4848v^2 + 390.4533v + 384 \\ 10.7240v^3 - 68.9430v^2 + 5.2191v + 170 \\ -14.3627v^3 + 47.3284v^2 - 25.9656v + 97 \end{bmatrix} \\
   \]

(a) (6 pts) Plot the two cubic planar parametric curves using $N$ equally distributed points in parameter space for three different values of $N$, $N = \{11, 101, 1001\}$. Draw both curves for each distinct $N$ value on the same plot.

(b) (4 pts) Plot the parametric speed $\frac{ds}{dt}$ of the two curves for the above three values of $N$, where $s = \text{arc length}$, $t$ is the parameter $u, v$, and

1
\[
\frac{ds}{dt} = \sqrt{x'^2(t) + y'^2(t) + z'^2(t)}
\] (1)

(c) (2 pts) Approximate the length of the two curves using polygonal approximation for the above three values on \( N \). (The polygonal approximation assumes that the length of the curve is equal to the sum of the lengths of straight line segments between consecutive points on the curve).

(d) (6 pts) Compute the tangent vectors \( \mathbf{r}(t) = \frac{\mathbf{d}}{\mathbf{d}t} \) at \( u = 0 \) and \( v = 0 \) and 1 for the two curves and compute the angle between the two tangent vectors at \( u = 1 \) and \( v = 0 \).

(e) (2 pts) Find the locations where \( y_1(u) \) and \( y_2(v) \) become maximum.

(f) (3 pts extra credit) Plot the curvature \( \kappa \) in terms of parameter \( u, v \) for the two curves for \( N = 101 \). If you are comfortable using MATLAB, feel free to calculate this value numerically from \( \mathbf{r}(t) \) without evaluating the following equation analytically for the given \( \mathbf{r}(t) \).

\[
\kappa = \frac{\mathbf{r} \times \mathbf{r} \cdot (\mathbf{r} \times \mathbf{r})}{(\mathbf{r} \cdot \mathbf{r})^3}
\] (2)

(g) (3 pts extra credit) Using the numerical values calculated in question (2f) estimate the parametric values which give absolute maximum and minimum of curvature for both curves. (Write a program in MATLAB using an algorithm for finding the maximum and minimum of \( N \) numbers for \( N = 101 \). Do not use the ‘max’ and ‘min’ functions in MATLAB).