13.003
Computational Geometry and Visualization

Problem Set 5

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Out: March 6, 2000
Due: 5:00pm March 17, 2000

1. (4 Pts) Show that the derivative of a Bézier curve (also called hodograph) of the form:

\[ \mathbf{R}(t) = \sum_{i=0}^{n} \mathbf{R}_i B_{i,n}(t) \quad 0 \leq t \leq 1 \]  \hspace{1cm} (1)

is given by:

\[ \mathbf{R}'(t) = \sum_{i=0}^{n-1} n(\mathbf{R}_{i+1} - \mathbf{R}_i) B_{i,n-1}(t) \quad 0 \leq t \leq 1 \]  \hspace{1cm} (2)

Sketch a cubic Bézier curve and its hodograph and their control polygons.

2. (4 Pts) Show how an explicit polynomial curve \( y = y(x) \), where \( a \leq x \leq b \) can be converted into a Bézier curve. Provide the control points of the resulting Bézier curve. Apply this to the case \( y(x) = (1 - x)^3, a = 0, b = 1 \). Hint: You need to use the linear precision property with \( x = t, t = \sum_{i=0}^{3} \frac{1}{3} B_{i,3}(t) \).

3. (4 Pts) Recall from Problem Set 1 that the sheer line (edge of deck) of our yacht tender is represented by two cubic planar parametric curves \( \mathbf{r}_1(u) = \{x_1(u), y_1(u), z_1(u)\} \) and \( \mathbf{r}_2(v) = \{x_2(v), y_2(v), z_2(u)\} \), where \( 0 \leq u, v \leq 1 \):

\[
\begin{align*}
\mathbf{r}_1(u) & = \begin{bmatrix} x_1(u) \\ y_1(u) \\ z_1(u) \end{bmatrix} = \begin{bmatrix} -105.3372u^3 + 257.0999u^2 + 232.2372u \\ 57.0275u^3 - 279.1296u^2 + 392.1021u \\ 38.7212u^3 - 19.1985u^2 - 92.5227u + 170 \end{bmatrix} \\
\mathbf{r}_2(v) & = \begin{bmatrix} x_2(v) \\ y_2(v) \\ z_2(v) \end{bmatrix} = \begin{bmatrix} -2.9684v^3 - 3.4848v^2 + 390.4533v + 384 \\ 10.7240v^3 - 68.9430v^2 + 5.2191v + 170 \\ -14.3627v^3 + 47.3284v^2 - 25.9656v + 97 \end{bmatrix}
\end{align*}
\]
(a) Convert these curves to Bézier form. You may choose to either use matrix operations or the end conditions (position and 1st derivative) to determine the control polygon of the curves.

(b) Plot the control polygons of the two curves in OpenGL with the profiles.

(c) Plot the convex hulls of the two curves in OpenGL with the profiles.

Hand in your plots and submit your code electronically.

4. (8 Pts) Draw a unit cube centered at the origin, with any of its faces parallel to one of the coordinate planes. Do the following transformation:

- Set the viewpoint at (5.0,5.0,5.0), and then rotate the cube about the view direction (vector between viewpoint and the center of the object) by 10°. Use orthographic projection first, and then change to perspective projection.
- For the following three transformations, different sequences result in different objects. Try the sequences a-b-c, b-c-a, and c-a-b, and turn in your plots:
  (a) Scale the x-axis of the modeling space by 2.
  (b) Rotate the modeling space first around the z-axis by 30°, and then around the y-axis by 45°.
  (c) Translate the modeling space by (1, 1, 1).

A skeleton program called xform_cube to get you started is provided in the course locker, /mit/13.003/ProblemSets/PS5.

Hand in your plots illustrating the different sequences and submit your code electronically.