13.003
Computational Geometry and Visualization

Extra Homework

Individual effort

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1. The following 6 control points define 3 segments of a uniform B-spline curve.

\[ V_0 = [0, 3], \ V_1 = [2, 5], \ V_2 = [4, 5], \ V_3 = [5, 4], \ V_4 = [5, 1], \ V_5 = [3, 0] \]

(a) Using equations for the position and the first derivative, find the Bézier representations for each of the 3 spans of this B-spline curve.

(b) Determine areas of the convex hulls for each of the 3 spans of the B-spline curve, and the three Bézier curves. Find the ratios of the areas of the convex hulls for each segment.

(c) Find the Ferguson and the Lagrange forms for the middle span.

(d) Find the tangent and normal vectors and the curvature at the parametric midpoint of the middle span.

2. Find the arc length of the epicycloid curve \( \{x(\theta), y(\theta)\} \), where \( 0 \leq \theta \leq \pi/2 \),

\[
x(\theta) = (r_0 + r)\cos(\theta) - r\cos\left(\frac{r_0 + r}{r}\theta\right),
\]

\[
y(\theta) = (r_0 + r)\sin(\theta) - r\sin\left(\frac{r_0 + r}{r}\theta\right).
\]

3. Find the parametric equation for the curve of intersection between the \( xy \) plane and the tangent lines to the helix curve \( \mathbf{r}(t) = \{\cos(t), \sin(t), t\} \), where \( t > 0 \).

4. Find the principal, Gauss and mean curvatures of the torus

\[
\mathbf{r}(\phi, \theta) = \{(b + a\sin(\phi))\cos(\theta), (b + a\sin(\phi))\sin(\theta), acos(\phi)\},
\]

as a function of \( \phi \) and \( \theta \).