13.013 Problem Set 11 Due Dec. 6, 2000

1. Express the complex number $5 + 2i$ in $Ae^{i\theta}$ and $Be^{-i\phi}$

2. Express $\frac{1}{3+5i}$ in $Be^{-i\phi}$ form.

3. Let $F(t) = (2 + 3i)e^{i\omega t}$ express the real part of $F(t)$ in the form as $A\cos(\omega t - \phi)$.

4. a) A simple dashpot is driven by a force $f(t) = f_0 \cos(\omega t)$. Assume the response is of the form $x(t) = x_0 \cos \omega t - \phi$ and find $x_0 + \phi$. . (Solution provided as an example.)

$$\sum F = M\ddot{x} = f_0 \cos(\omega t) - R\dot{x} = 0 \Rightarrow f_0 \cos(\omega t) = R\dot{x}$$

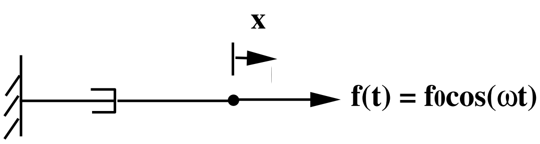

Figure 1:

assume $x(t) = x_0 cos(\omega t - \phi) \Rightarrow \dot{x} = -\omega x_0 \sin(\omega t - \phi)$ therefore, $f_0 \cos(\omega t) = R\dot{x} = -R\omega x_0 \sin(\omega t - \phi)$. The previous equation is only satisfied when $\phi = \pi/2$, i.e. $\sin(\omega t - \frac{\pi}{2}) = -\cos(\omega t)$ and $f_0 \cos(\omega t) = R\omega x_0 \cos(\omega t)$ $f_0 = R\omega x_0$, $x_0 = \frac{f_0}{R\omega}, \phi = \pi/2$

b) Find ϕ and x_0 for the case of a simple spring.

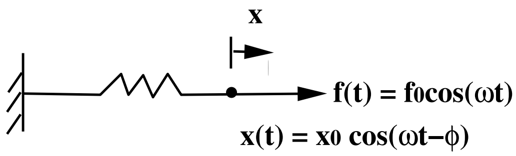

Figure 2:

c) Do the same for a simple mass find x_0 and ϕ

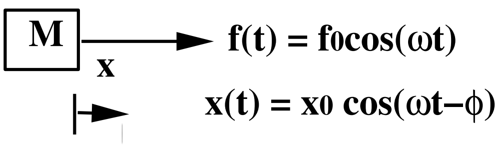

Figure 3:

5. Do problem 8-82 in the text.

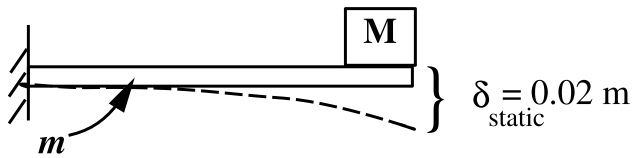

Figure 4:

6. A cantilever beam of mass M deflects $0.02m$ in the vertical direction when a concentrated mass, is placed at the tip as shown

a) What is the natural frequency of the system assuming it behaves like a standard SDOF system? Is this estimate likely to be high or low? Use only data given in part a).

b) Given $\xi = 0.02$, $F_0 = 0.05Mg$ and $\frac{\omega}{\omega_n} = 1.0, 5.0$ and 0.5 Compute the magnitude and phase angle of the response, x for the 3 values of $\frac{\omega}{\omega_n}$ specified.

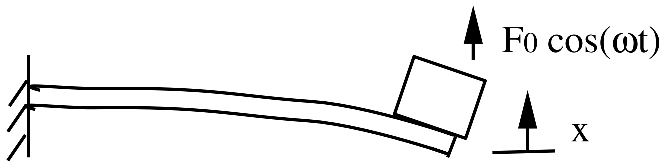

Figure 5:

7. Let $\frac{\omega}{2\pi} = 20Hz$ and $M = 1kg$

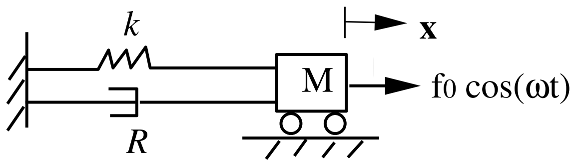

Figure 6:

a) Design the spring support, k , such that the maximum dynamic response

i) $|x| = 0.1\frac{f_0}{k}$

ii) what is the required value of $\frac{\omega}{\omega_n}$?

iii) what ξ have you assumed?

b) If the excitation frequency occasionally changes such that $\omega = \omega_n$ then what value of damping ratio must you have so that $\frac{x}{x_{static}} \leq 20.$?

Useful trig relations from appendix H-6 iin the text and other texts.
Complex numbers: $a + bi = ce^{i\theta}$
$c = \sqrt{a^2 + b^2}$, $\quad \theta = \arctan(\frac{b}{a})$

or $a + bi = ce^{-i\phi}$
$c = \sqrt{a^2 + b^2}, \quad \phi = -\arctan(\frac{b}{a})$
useful trig formula: If

$$
\begin{aligned}
x(t) &= c\cos(\omega t - \phi) \\
&= c[\sin\omega t \sin\phi + \cos\omega t \cos\phi]
\end{aligned}
$$

or if

$$
\begin{aligned}
x(t) &= c\sin(\omega t - \phi) \\
&= c[\sin\omega t \cos\phi - \cos\omega t \sin\phi]
\end{aligned}
$$