MIT Department of Ocean Engineering
13.013 Dynamics and Vibration Quiz I, Fall 2000
Monday, Octover 23, 2000

Read all questions first
INDIVIDUAL EFFORT

Problem 1:

A mass m slides frictionlessly on a semicircular track of radius R, as sketched in the figure below, starting from rest at $\theta = 0$. Note that gravity g acts.

a) (15 points) Determine the contact force as a function of θ only (ie. not of $\dot{\theta}$), sketch the function and give a physical explanation for its values at $\theta = 0, \pi/2$ and π.

b) (10 points) Determine the time period T it takes for the particle of mass m to return to the initial position $\theta = 0$ expressing it in terms of an integral. Explain why the mass returns to $\theta = 0$.

c) (5 points) Is the above time period (in question b) finite? (HINT: No need to attempt to fully evaluate the integral).

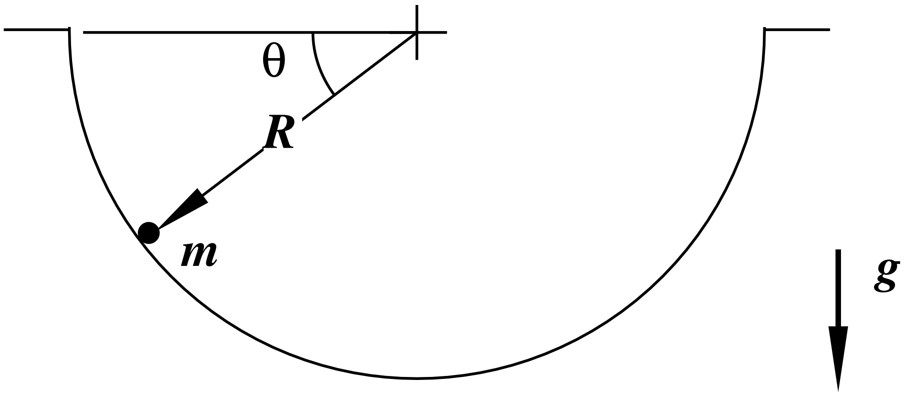

Figure 1:

Problem 2:

A soft ball of mass m traveling horizontally hits a vertical rigid wall at a speed of v_0 and returns at half that speed. The impact lasts a time period T and the contact force can be approximated by:

$$F(t) = F_{max} sin(\pi t/T)$$

a) (10 points) Determine F_{max} as a function of v_0, m, T.

b) (10 points) Determine the velocity $v(t)$ of the ball as a function of t, sketch the function, and explain its slope at $t = 0$, $T/2$, T.

c) (15 points) Determine the kinetic energy loss in the process of impact and the work of the force $F(t)$ and prove that they are equal.

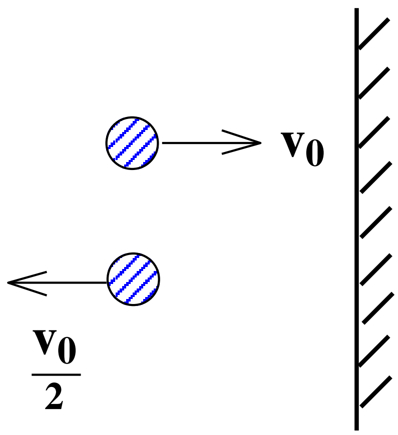

Figure 2:

Problem 3:

A wheel of radius r is rotating at $\omega_1(t)$ with respect to a platform. The platform is rotating at $\omega_2(t)$ with respect to ground, as sketched in the figure below.

Let B be a tiny bug of mass m which lies along a vertical radius of the wheel, at the instant shown.

Consider a rotating frame $Oxyz$ rigidly attached to the vertical rotating wheel, as in the figure. The bug B is along the $0z$ axis at the instant shown.

a) (10 points) Find the velocity of point B with respect to the ground and interpret it physically.

b) (15 points) Find the acceleration of point B with respect to ground and interpret each term physically.

c) (10 points) Note that gravity g acts. Find the force (vector) from the vertical rotating wheel on the bug B at the instant shown.

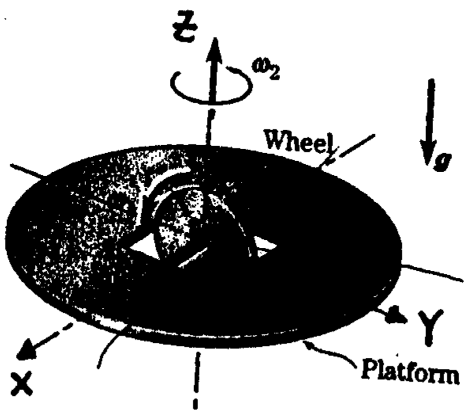

Figure 3: