MIT Department of Ocean Engineering
13.013 Dynamics and Vibration Quiz II, Fall 2000
Monday, November 20, 2000

Read all questions first
INDIVIDUAL EFFORT

Problem 1:

In the mass spring dashpot system sketched in the figure below, mass m moves without friction on a horizontal floor. It is restrained by a spring of spring constant k and a dashpot of dashpot constant c connected in series to a rigid wall. A force *F(t)* acts on the mass.

a (20 points) Derive the system of two differential equations of motion involving displacements x and y as the figure below. Confirm your answer by using both the direct (10 points) and the variational method (10 points).

b (10 extra credit) Eliminating y and $\dot{y}$, derive the differential equation of motion involving only displacement x. (HINT: The resulting equation involves the 3rd derivative of x with respect to time t.)

c (10 points) Assume that $c = 0$ and the system reduces to a single degree of freedom system for $x(t)$. Another mass m with velocity $-v_0\hat{i}(v_0 > 0)$ hits the mass m attached to the spring at $t = 0$ and sticks to the mass. Find the resulting velocity of the combined mass immediately after the collision. Finally determine the maximum compression and extension of the spring and when these occur for the first time after $t = 0$.

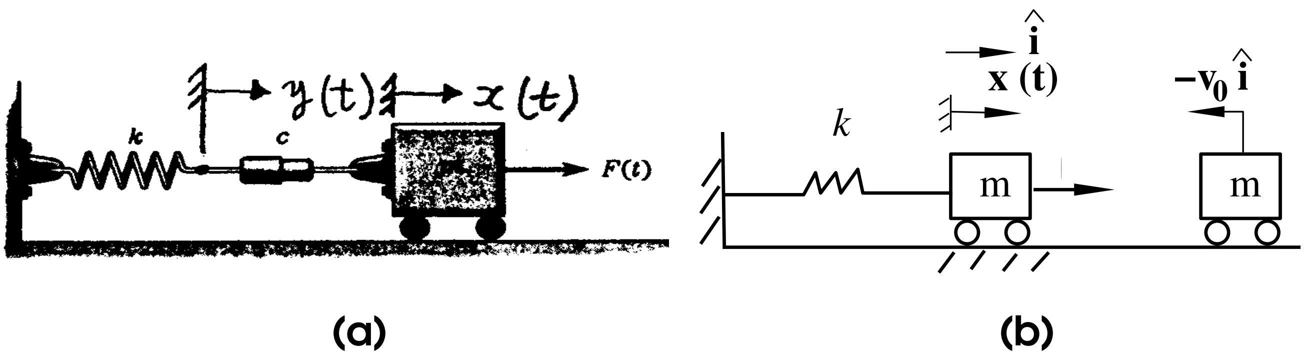

Figure 1:

Problem 2:

The system shown in the figure below is made of four rigid massless rods each of length a which are restrained to move on the same vertical plane (pivoting at point A which is fixed in an inertial frame without friction). Two masses m_1 and m_2 are attached to the rods as in the figure. Mass m_2 can move along the vertical (inertial) axis Ay that passes through point A. The friction of mass m_2 along Ay can be modelled as a dashpot with constant c. The whole system has an *imposed* rotation about Ay with a constant angular velocity Ω. Gravity g acts along Ay.

a (20 points) Explain why the angle θ forms a complete and independent set of generalized coordinates for the system and prove that the Lagrangian function of the system is given by

$$L = m_1 a^2(\dot{\theta}^2 + \Omega^2 \sin^2\theta) + 2m_2 a^2 \dot{\theta}^2 \sin^2\theta + 2ga(m_1 + m_2)\cos\theta$$

b (10 points) Determine the generalized force on the system by computing the virtual work under an admissible variation $\delta\theta$.

c (10 points) Using Lagrange's equation, derive a second order non-linear differentail equation of motion of the system.

d (5 points) Set $\theta(t) = \theta_s + \phi(t)$ where θ_s is the static equilibrium position and determine the two possible values of θ_s.

e (15 points) Linearize the equation of motion for small angles $\phi(t)$ around each of the two static equilibrium angles to obtain two linearized equations of motion.

f (10 points) If the dashpot constant $c = 0$, derive conditions for which each of the linearized differential equations in question (e) has stable solutions.

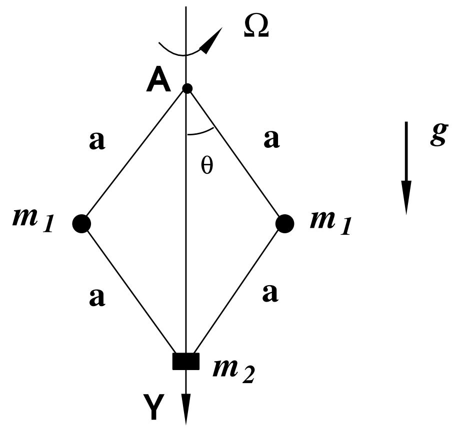

Figure 2: