MIT Department of Ocean Engineering
13.013 Dynamics and Vibration
Quiz III, Fall 2000
Monday, December 18, 2000

9am to noon, Room 5-134

Read all questions first

OPEN BOOK: Textbooks by Williams and/or Rao, homework and your own notes

INDIVIDUAL EFFORT

Problem 1: *(34 points)*

A system of two rigid slender rods of mass m and length l are pivoting on a plane as in Figure 1, around point O (fixed in an inertial frame) and point A both without friction. Point B is constrained to move along the x axis also without friction. Points O and B are connected with a linear spring of constant k which is at its unstretched length $(2l\cos\phi_0)$ when the angle $\phi = \phi_0$. There is *no gravity* acting on this problem.

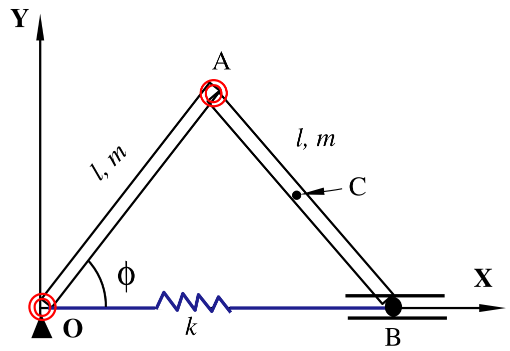

Figure 1:

a) *(14 points)* Explain why the angle ϕ is an appropriate generalized coordinate and show that the Lagrangian L of the system is

$$L = \frac{ml^2}{3}(1 + 3\sin^2\phi)\dot{\phi}^2 - 2kl^2(\cos\phi - \cos\phi_0)^2$$

b) *(5 points)* Using Lagrange's equation (*indirect* method), show that the second order differential equation of motion of the system is:

$$2(1 + 3\sin^2\phi)\ddot{\phi} + 3\dot{\phi}^2\sin 2\phi - 12\lambda^2(\cos\phi - \cos\phi_0)\sin\phi = 0, \textit{where } \lambda = (k/m)^{1/2}$$

c) *(15 points)* Now using the *direct* method formulate the equations of motion of the system as follows:

c1: Create the free-body diagram for each rigid rod.

c2: Write the equations for the rate of change of linear momentum for each rod.

c3: Write the equations for the rate of change of angular momentum for each rod (*Hint*: For rod OA take moments about O and for rod AB take moments about its center of mass C with coordinates $X_c = \frac{3l}{2}\cos\phi, Y_c = \frac{l}{2}\sin\phi$).

c4: Write the constitutive law for the spring.

c5: Verify that you have enough equations to solve for the unknowns, but *do not* attempt to solve the resulting system to get a single differential equation in terms of ϕ (as in question b) by eliminating the forces or center of mass displacements involved in the free body diagrams. (This is meant to illustrate the convenience of the indirect method).

<u>Problem 2</u>: *(33 points)*

A mortar shell is shot almost vertically from a rail car. The rail car is spring supported as shown in Figure 2. It also has a damper as shown. The mass of the car and mortar launcher is $10,000$ kg. The mass of the shell is 100 kg. The velocity of the shell is 300 m/s as it leaves the mortar. The spring stiffness of the suspension system of the rail car is $k = 15,791$ N/m. The damper constant, R, is 251.33 N/(m/s).

Hint: I suggest that you use a coordinate system corresponding to the at rest position of the car after the vibration has stopped.

a) *(10 points)* Write down the equation of motion of this single degree of freedom system for vertical motion. Determine the natural frequency and damping ratio (OBTAIN NUMERICAL VALUES).

b) *(13 points)* The mortar is loaded and fired when the system is initially at rest, (*no motion*). After firing the shell the car will vibrate. Find an expression for $x(t)$, the time history of vibration of the car, after the shell is fired. Be specific about the initial conditions that you use. Sketch the result.

c) *(10 points)* The vibration of the car will decay with time after firing. Predict the ratio of two vibration amplitude peaks separated by five periods of vibration.

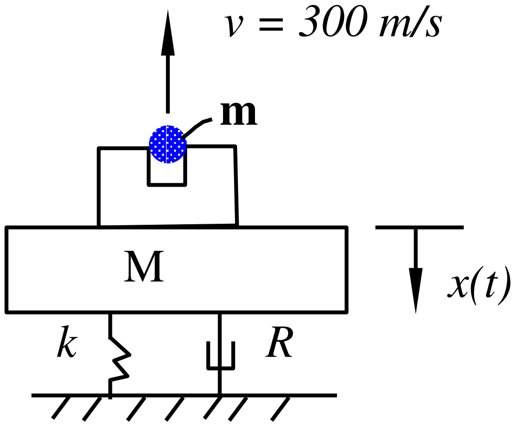

Figure 2:

Problem 3: *(33 points)*

You own a pump that is causing a lot of vibration in your lab. You decide that a rotating imbalance inside of the pump is causing a harmonic force to be applied vertically to the bench on which it sits. In an attempt at a quick fix, you decide to mount the pump on a lightweight (compared to the pump) cantilever as shown in Figure 3(a). You set the machine on the cantilever and turn it on. You experiment by sliding the pump at various positions along the cantilever until you reach a position at which the beam vibrates quite violently. This position is three quarters of the length of the cantilever. At this location the vibration is greater than at any other position on the cantilever.

a) *(5 points)* Your lab partner wants to know what is going on. Explain why the system vibrates violently at this location. No math, just a few sentences describing the phenomenon.

b) *(14 points)* You turn off the machine and measure the change in the static deflection of the cantilever at the location of the machine, when you lift the machine off of the cantilever. The deflection is 0.01m.

i. What is the natural frequency of the beam with the pump located at that position?

ii. What is the frequency of the force produced by the machine?

c) *(14 points)* When the pump is located at the end of the beam (see Figure 3-(b)) the natural frequency is 0.42 times the frequency of the pump

$\omega_n = 0.42\omega$, where ω is frequency in radians/second.

i. Predict the ratio of the magnitude of the force transmitted to the structure at the base of the cantilever to the excitation force produced by the pump. Assume very light damping to make the estimate easy.

ii. If the damping increases would the transmitted force increase or decrease?

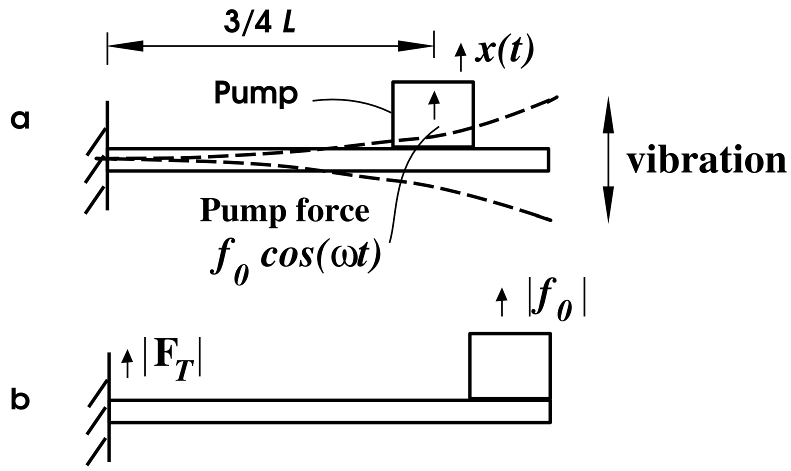

Figure 3: