13.013J/1.053J, Dynamics and Vibration, Quiz II
Wednesday, November 28, 2001; 11:00am-12:30pm

CLOSED BOOK AND NOTES, TWO SHEETS OF FORMULAS
INDIVIDUAL EFFORT. READ ALL PROBLEMS FIRST!!

Problem 1 (30 points):

Using the variational formulation determine the equations of motion for the system illustrated in Figure 1, where the masses m_1, m_2, the linear spring constants k_1, k_2 and the dashpot constant c, and the magnitude F_0 and frequency ω of the external force $F(t)$ are given.

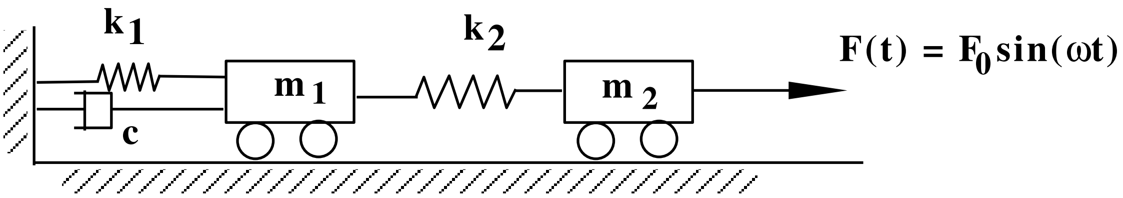

Figure 1: Problem 1

Problem 2 (30 points):

Using the direct formulation derive the non-linear differential equation of motion of the hoop-spring system shown in Figure 2 and then linearize it around the static equilibrium $\theta = 0$. When $\theta = 0$ the spring is unstretched. The hoop's mass and radius m, R; the spring constant k, force magnitude and frequency F_0, ω, and the acceleration of gravity g are all given data.

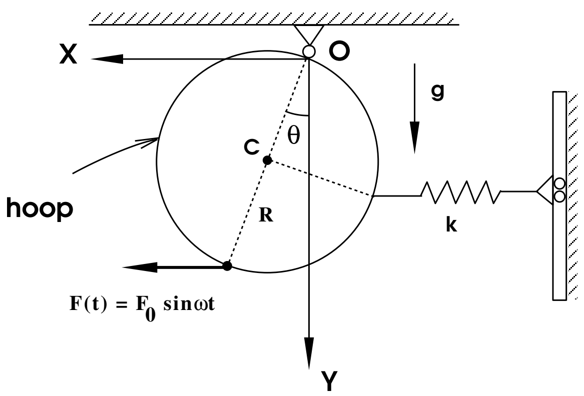

Figure 2: Problem 2

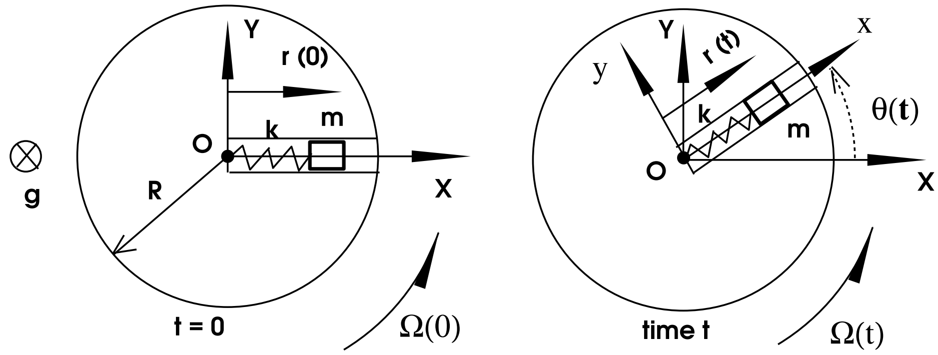

Figure 3: Problem 3

Problem 3 (40 points):

A turntable of radius R rotates in a horizontal plane (with respect to gravity) about a vertical axis passing through its center O, at a specified angular non-constant angular velocity $\Omega(t)$. Consider point O to be fixed in inertial space. A particle of mass m is attached to the center of the turntable by a massless spring of a spring constant k and slides without friction along a radial track on the turntable as in Figure 3, so that the distance of mass m from O is $r(t)$. The unstretched length of the spring is l_0. Assume that always $0 < r(t) \leq R$, and that $l_0 < R$.

a) (20 points) Using the variational formulation, show that the equation of motion of the mass m is the following second order ordinary differential equation for $r(t)$: $m\ddot{r} + [k - m\Omega^2(t)]r = kl_0$.

b) (5 points) Assume $\Omega(t) = \omega_0$, a constant such that $k > m\omega_0^2$. Find the static equilibrium distance r_0 of the mass m from O.

c) (5 points) Assuming small motions of the mass m around r_0 defined in question (b), i.e. setting $r(t) = r_0 + \varepsilon(t)$ where $\varepsilon(t)$ is small, show that under the condition of question (b), the position $r = r_0$ is a stable equilibrium position.

d) (10 points) Describe physically what happens if the turntable and mass start from rest at $t = 0$ and then suddenly at $t = 0$ the turntable is forced to rotate at a constant $\Omega(t) = \omega_1$ such that $mw_1^2 > k$. Determine the solution of the equation of motion for $r(t)$ in this case. Explain how you would find after how much time the equation derived in (a) becomes invalid? (but do not actually carry out the algebra).