MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF OCEAN ENGINEERING

AND

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

13.013J/1.053J Dynamics and Vibration

Fall 2001

Quiz 1 (Make-up)

2:30am – 4:30pm

Friday, October 26, 2001

CLOSED BOOK & NOTES, ONE SHEET OF FORMULAS

FIRST, READ ALL PROBLEMS!

INDIVIDUAL EFFORT

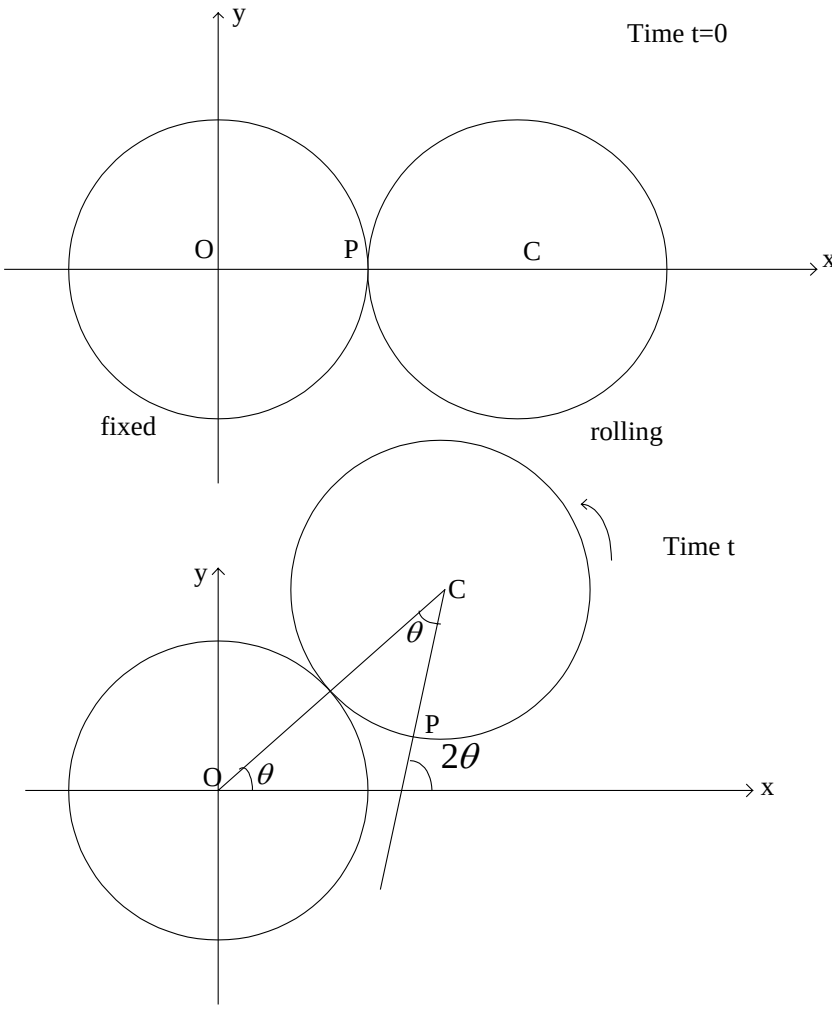
Problem 1: (40 points)

A frictionless rectilinear tube AB rotates with uniform angular velocity $\vec{\omega} = \omega\vec{k}$ in a plane C normal to the unit vector $\vec{k}$. Let O be the middle of the tube. Point O of the tube has a uniform acceleration a_0 along a fixed straight line on the plane C starting from rest of time t=0. At time t=0, a particle of mass m is released at point O inside the tube.

(a) (15 points) Let *r(t)* be the distance of the particle from O at time *t*. Find expressions for the inertial velocity and acceleration of the particle.

(b) (20 points) Using Newton's laws, derive a single second order differential equation of motion for *r(t)*.

(b) (5 points) What are the initial conditions for *r(t)* needed for the solution of the ordinary differential equation found in question (b)?



Problem 2: (40 points)

A circle of radius *a* rolls without slipping with a variable angular velocity on a fixed circle of the same radius *a*. Let O and C be the centers of the fixed and moving circles. Let θ be the angle between a fixed axis $\vec{OX}$ and $\vec{OC}$. At time *t*=0, a material point P on the moving circle is the contact point of the two circles also along the axis $\vec{OX}$.

(a) (15 points) Find an expression for the inertial velocity of material point P as a function of θ and $\dot{\theta}$.

Hint: Use the angle 2θ between axis *X* and $\vec{PC}$ as in the figure.

(b) (25 points) Show that the inertial accelerations of material point P along $\vec{CP}$ and normal to $\vec{CP}$ are

$$a_r = -2a\left(2\dot{\theta}^2 + \ddot{\theta}\sin\theta - \dot{\theta}^2\cos\theta\right)$$

$$a_n = 4a\sin\frac{\theta}{2}\left(\ddot{\theta}\sin\frac{\theta}{2} - \dot{\theta}^2\cos\frac{\theta}{2}\right)$$

Hint: Use the formulas $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$, $\cos\theta = 1 - 2\sin^2\frac{\theta}{2}$

Problem 3: (20 points)

A particle of mass m_1 is at a height h above the static equilibrium of a mass m_2 which is supported by a spring of constant k to the ground. Mass m_1 falls and sticks to mass m_2. Find the maximum downward deflection of the combined mass m_1 and m_2.