MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF OCEAN ENGINEERING

AND

DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

13.013J/1.053J Dynamics and Vibration

Fall 2001

Quiz 1

11:00am – 1:00pm

Friday, October 19, 2001

CLOSED BOOK & NOTES, ONE SHEET OF FORMULAS

FIRST, READ ALL PROBLEMS!

INDIVIDUAL EFFORT

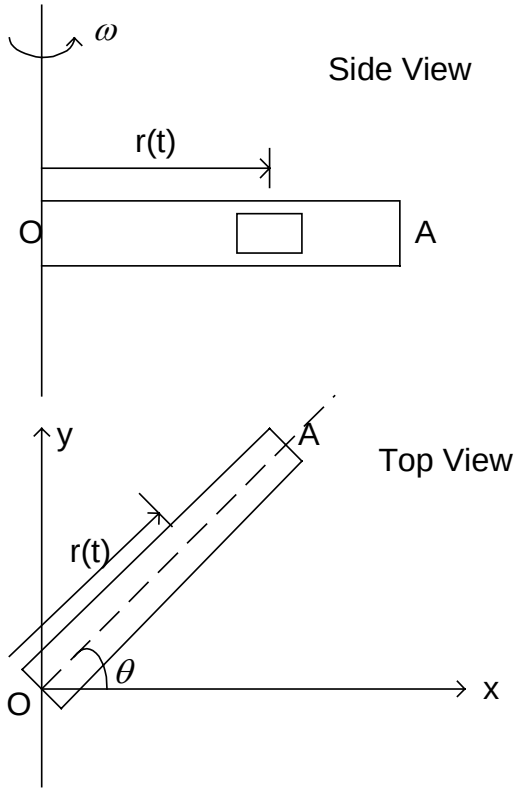
Problem1: (40 points)

A straight tube OA is made to rotate in a plane round a fixed point O with uniform angular velocity $\vec{\omega} = \omega\vec{k}$, where $\vec{k}$ is a unit vector normal to the plane. The tube contains a particle of mass *m*. Let *r(t)* be the distance of the particle from O. At time $t = 0$, the particle is released in the tube a distance *a* from point O. Neglect gravity effects and assume that the kinetic friction coefficient is μ.

(a) (15 points) Find expressions for the inertial velocity and acceleration of the particle, and identify by name each of the terms of the resulting equations.

(b) (20 points) Using Newton's laws, derive a single second order differential equation of motion for *r(t)*.

(c) (5 points) What are the initial condition for *r(t)* needed for the solution of the ordinary differential equation found in question (b) ?

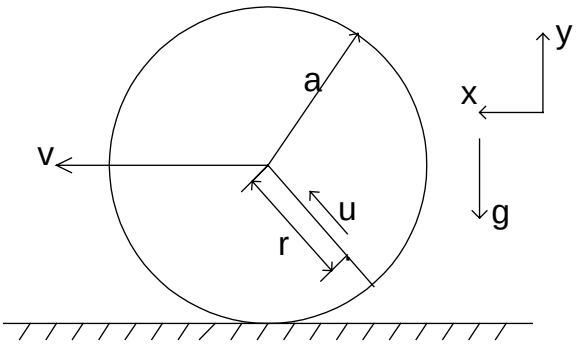


Problem 2: (30 points)

An insect of mass *m* crawls at a constant rate (speed) *u* along the spoke of a cartwheel of radius *a* towards the center, the cart moving with velocity *v* along a horizontal direction *X* without slipping of its wheels. The acceleration of gravity is g.

(a) (15 points) Find the inertial velocity and inertial acceleration of the insect along and normal to the spoke in terms of *u*, *v*, *a* and *r* (the distance of the insect from the center of the wheel).

(b) (15 points) Find the force from the spoke on the insect.



Problem 3: (30 points)

A body of mass $2m$ moving with speed u along a straight line in an inertial frame is split into two parts of equal mass m by an internal explosion which generates kinetic energy E. If the two masses after the explosion move on the same straight line as before, show that the absolute value of their relative speed is $2\sqrt{\frac{E}{m}}$ and find their actual speeds.