MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF OCEAN ENGINEERING AND
CIVIL AND ENVIRONMENTAL ENGINEERING

13.10J/1.573J Structural Mechanics

Fall 2001

Final Exam

9:00am - noon

Wednesday, December 19, 2001
Room 3-370

OPEN BOOK & NOTES
FIRST, READ ALL PROBLEMS !!!
INDIVIDUAL EFFORT

1. (20 pts) A cylindrical beam of diameter d, length L, Young's modulus of elasticity E and specific weight γ is pin-supported at B and placed against a smooth (frictionless) wall at A. If no buckling happens, answer the following five questions:

(**Hint** : Due to the weight, the beam will experience bending and axial force. Bending moments resulting from the axial force due to deflection may be neglected.)

(a) (4 pts) Find the reaction force at A

(b) (4 pts) Express the axial force as a function of x.

(c) (4 pts) Express the bending moment as a function of x.

(d) (4 pts) Express the maximum compressive stress as a function of x.

(e) (4 pts) Find the position along x where the maximum compressive stress occurs.

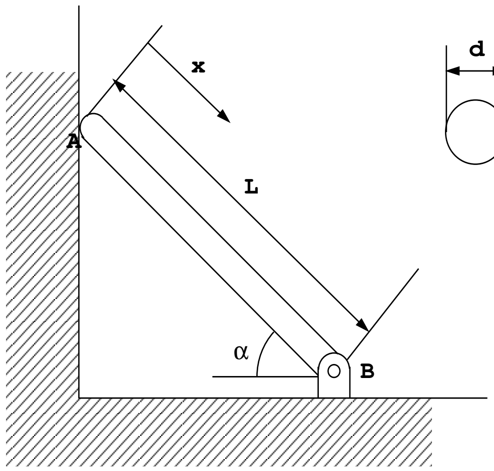

Figure 1: Problem 1

2. (15 pts) A cylindrical shaft consisting of two parts of length $\frac{L}{2}$ each having two different polar moments of inertia J_a and J_b ($J_a > J_b$) and shear modulus of rigidity G is fixed at A and B. If a torque T is applied at distance x from A, determine x so that the torsional reactions at A and B become equal.

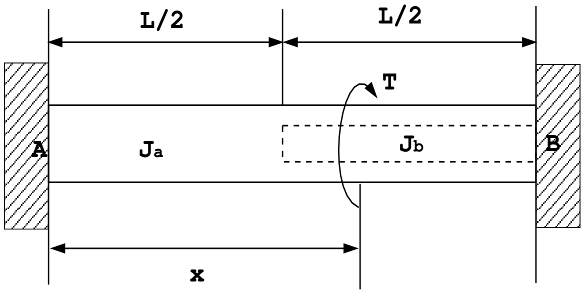

Figure 2: Problem 2

3. (20 pts) A slender curved circular cross section beam with constant bending rigidity EI is fixed at one end. The beam is a quarter of the circle with radius of R and is subject to moment M_o at 45^o whose direction is normal to $x-y$ plane as shown in Figure 3. Using the energy method and considering bending effects only, determine:

(a) (7 pts) The horizontal deflection δ_h at the free end.

(b) (6 pts) The vertical deflection δ_v at the free end.

(c) (7 pts) The angle of rotation θ at the free end.

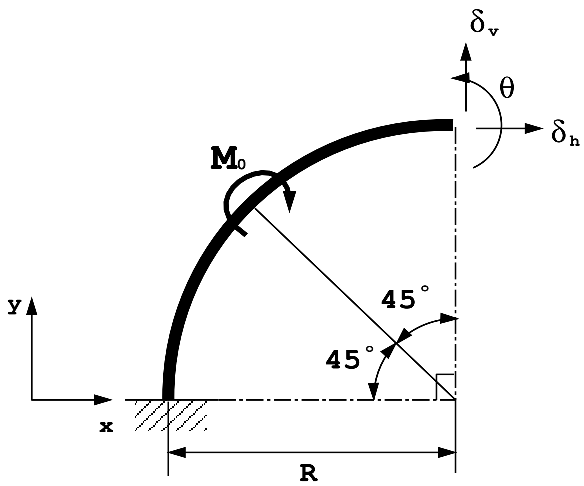

Figure 3: Problem 3

4. (20 pts) A structure in the form of a three-sided rectangular shape lies in a horizontal plane, has both ends clamped, and is subject to a vertical force P as shown in Figure 4. The structure is made of a slender beam with a constant circular cross section of radius r and has Young's modulus of elasticity E and shear modulus of rigidity G.

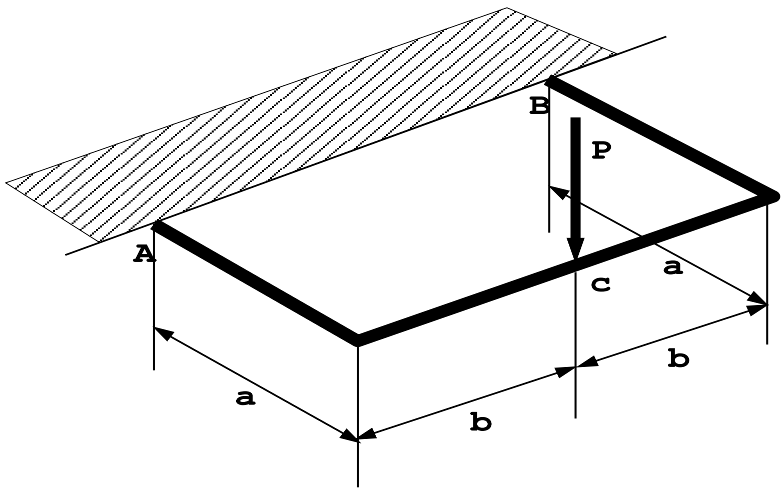

Figure 4: Problem 4

Using the energy method and considering only bending and torsion effects:

(a) (15 pts) Determine the reactive torque at the supports, A and B.

(b) (5 pts) Find the vertical deflection at C under the load P.

5. (25 pts) Answer the following questions.

(a) (10 pts) A weightless cylindrical beam of length L, bending rigidity EI and cross sectional area S is installed with fixed supports at A and B at temperature T as shown in Figure 5. The material has a linear coefficient of thermal expansion α. Using the differential equation formulation, determine the increase in temperature ΔT at which elastic instability (buckling) first occurs.

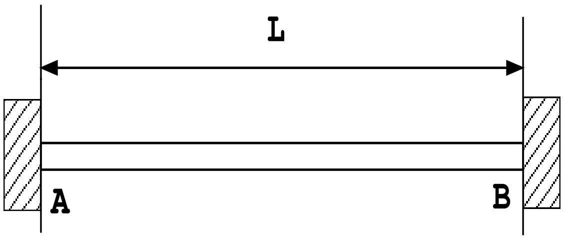

Figure 5: Problem 5-1

(b) (15 pts) If the beam has a varying cross section of a circular shape with a varying bending rigidity $EI(x) = EI_A + (EI_B - EI_A)\frac{x}{L}$ as shown in Figure 6, using the trial function $y(x) = A(1 - \cos(\frac{2\pi}{L}x))$ for approximating the deflection, derive an approximate analytical expression for the critical temperature increase ΔT at which elastic instability (buckling) first occurs using the energy method.

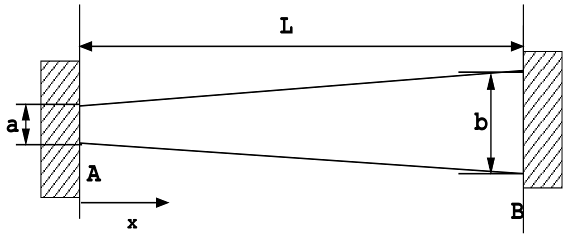

Figure 6: Problem 5-2

Note:

$$\begin{aligned}
\int x\cos(ax)dx &= \frac{1}{a^2}(\cos(ax) + ax\sin(ax)) \\
\int x\sin(ax)dx &= \frac{1}{a^2}(\sin(ax) - ax\cos(ax))
\end{aligned}$$