

MASSACHUSETTS INSTITUTE OF
TECHNOLOGY
DEPARTMENT OF OCEAN ENGINEERING
AND
CIVIL AND ENVIRONMENTAL ENGINEERING
13.10J/1.573J Structural Mechanics
Fall 2001

Quiz 1

2:00pm - 3:00pm
Friday, October 5, 2001
Room 3-370

**CLOSED BOOK & NOTES, ONE SHEET
OF FORMULAS
FIRST, READ ALL THREE PROBLEMS !!!
INDIVIDUAL EFFORT**

1. (30pts) Two forces F and $3F$ are applied to the rigid bracket BCD as shown in Figure 1.
 - (a) (10pts) Knowing that the cylindrical control rod AB is to be made of steel having an ultimate normal stress of σ_{nu} , determine the diameter d_{AB} of the rod for which the factor of safety with respect to failure will be f with $f > 1$.
 - (b) (10pts) The pin at C is to be made of steel having an ultimate shearing stress of σ_{su} . Determine the diameter of the pin C for which the factor of safety with respect to shear will be f .
 - (c) (10pts) Determine the required thickness t of the bracket supports at C knowing that it is made of steel having an ultimate normal stress of σ_{nu} and the factor of safety is f .

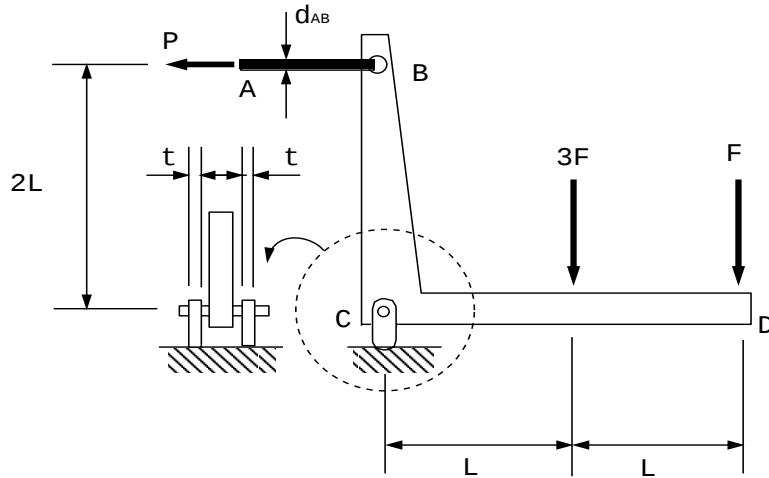


Figure 1: Problem 1

2. (35pts) The stress tensor at a given point in the XYZ coordinate system is:

$$\tau_{ij} = \begin{bmatrix} 100 & 400 & 0 \\ 400 & -200 & 100 \\ 0 & 100 & 0 \end{bmatrix} Pa \quad (1)$$

- (a) (15pts) A new frame $X'Y'Z'$ is formed by rotating the reference XYZ . Find the unknown elements of the stress tensor below in the $X'Y'Z'$ system:

$$\tau'_{ij} = \begin{bmatrix} 371.41 & \tau_{x'y'} & \tau_{x'z'} \\ 70.10 & \tau_{y'y'} & 86.6 \\ 50.00 & \tau_{z'y'} & \tau_{z'z'} \end{bmatrix} Pa \quad (2)$$

- (b) (5pts) Write down the equation for finding the principal stresses (but do not solve it).
- (c) (15pts) If the transformation from XYZ to $X'Y'Z'$ involves only rotation about the Z axis (i.e. $Z = Z'$), determine the angle of rotation and construct the rotation tensor Ω .

3. (35pts) The following displacement field describes the movement of a body under load in the XYZ coordinate system:

$$u = [(x^2 + 3), (3y^2z), x(1 - z)] \times 10^{-2}m \quad (3)$$

Let $P = (0, 3, 1)$ be a point within the above body.

- (a) (5pts) Determine the strain tensor at point P .
- (b) (9pts) Compute the normal strain at point P in the direction of unit vector $\vec{n}_1 = (0.6, 0.8, 0.0)$.
- (c) (9pts) Compute the shear strain at point P on a plane normal to the unit vector $\vec{n}_1 = (0.6, 0.8, 0.0)$ and along the unit vector $\vec{n}_2 = (0.8, -0.6, 0.0)$.
- (d) (12pts) Find the principal strains and the directions of the principal axes at point P .