 Quiz 2

6:00pm - 8:00pm
Thursday, November 15, 2001
Room 3-370

CLOSED BOOK & NOTES, TWO SHEETS OF FORMULAS
FIRST, READ ALL THREE PROBLEMS !!!
INDIVIDUAL EFFORT
1. (25pts) The normal strains at a point $P$ along directions 1, 2 and 3 on the planar surface of a linearly elastic thin plate with Young’s modulus of elasticity $E = 200 GPa$ and Poisson’s ratio $\nu = 0.3$ are measured as shown in Figure 1. Note that due to Poisson effect there may exist strain orthogonal to the $xy$ plane.

$$
\begin{align*}
\varepsilon_{xx} &= 2\mu \\
\varepsilon_{yy} &= -2.6\mu \\
\varepsilon_{y'y'} &= 1.3\mu 
\end{align*}
$$

Figure 1: Problem 1

(a) (15pts) Find $\varepsilon_{xy}$ and then find the principal strains and their directions.

(b) (10pts) Describe a method to determine the principal stresses and their directions but do not carry out the algebra.
2. (40pts) The structure shown in Figure 2 is proposed as a calibration fixture for an experiment. The cantilevered beam DF has an inverted “T-shape” rigid bracket attached to its tip F, as shown in Figure 2, allowing the placement of the point load $P$ at any location along ABC.

![Figure 2: Problem 2-a](image)

(a) (9pts) Assuming that the bending rigidity of the beam DF is $EI$, sketch the resulting shear force and bending moment diagrams for the beam DF for the load placed at A, B and C. Indicate the positions and values of the maximum bending moments for each case.

(b) (18pts) A point load $P$ is acting at C in the negative $y$ direction and another point load $P$ is acting at F in the positive $z$ direction, as shown in Figure 3.

i. (9pts) Find the neutral axis at the position where the maximum bending moment occurs.

ii. (9pts) Find the maximum normal stress $\tau_{xx}$. 

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(c) (23pts) The same beam structure with the same loads as shown in Figure 3 is considered. However, the cross section of the beam DF consists of two different materials as shown in Figure 4,

Figure 4: Problem 2-c

i. (10pts) Find the neutral axis at the position where the maximum bending moment occurs.

ii. (13pts) Find the maximum normal stress $\tau_{xx}$. 
3. **(10pts)** A rigid beam RT is supported by three steel rods as shown in Figure 5. A point load $P$ is applied at location $x$ on the beam RT. Assume that gravity effects can be neglected and the three bars display linearly-elastic behavior. Young’s modulus of elasticity $E$, the cross sectional area $A$ and the length $L$ of the three bars are indicated in the figure, respectively. While the three rods are elastic, find the position $x$ so that the beam remains horizontal.

![Figure 5: Problem 3](image-url)
4. (25pts) A rigid beam RT is supported by two steel rods as shown in Figure 6. A point force $P$ is acting downward at the position $x = \frac{8}{5}L$. The bars are idealized to be linearly-elastic, perfectly plastic as shown in Figure 6. The midpoint $Q$ is deflected $\frac{\tau y L}{3E}$ downwards by the slow application of the force $P$, after which the force is slowly removed. (Hint: Remember that when a material becomes plastic, no increase of force is required to get additional deformation. Also when the midpoint is deflected $\frac{\tau y L}{3E}$ it is NOT necessary that both rods are plastic.)

![Figure 6: Problem 4](image)

(a) (10pts) What is the required maximum value of $P$?

(b) (15pts) Find the final position of the beam after the force is removed.