Massachusetts Institute of Technology

Department of Ocean Engineering

Department of Civil and Environmental Engineering

Structural Mechanics 13.10J/1.573J

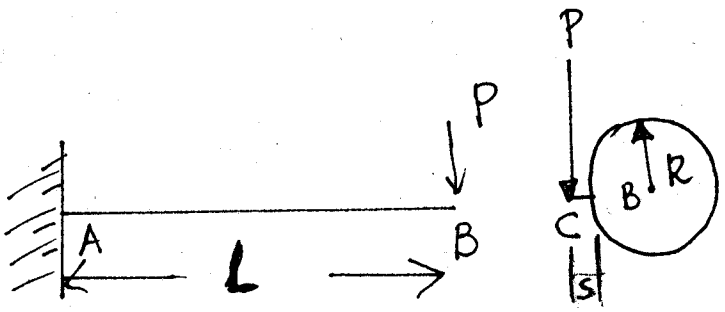
Final Examination
Fall 2002

Friday, December 20, 2002, 9am - noon

DO ALL FOUR PROBLEMS

OPEN BOOK AND NOTES
INDIVIDUAL EFFORT

1. **(25 pts)** The straight uniform material beam of length L given in Figure 1-(a) has a circular cross-section of radius R. It is subject to a vertical load P applied as shown in Figure 1-(b). If you are given the Young's modulus of elasticity E and the shear modulus G, calculate the vertical displacement at the point C (the length s is negligible) where the force P is applied. Shear deformation due to bending may be neglected.



(Figure 1-(a)) (Figure 1-(b))

Figure 1

2. **(25 pts)** A beam structure CAB made of linearly elastic uniform material of the Young's modulus of elasticity E as in Figure 2 is composed of a straight segment CA of length R and of a curved segment AB. The segment AB is a quarter of a circle of radius R and center O as in Figure 2. The cross-sections of the beam segments AB and CA are circular of radius r. The two beam segments are rigidly welded at point A. The end C is free and the end B is fixed (clamped to a vertical wall) and point A is supported on a pin-roller. A vertical force P is acting at the free end C as in Figure 2.

   (a) **(12 pts)** Determine the reaction force at A.

   (b) **(13 pts)** Determine the vertical deflection δ_v at point C.

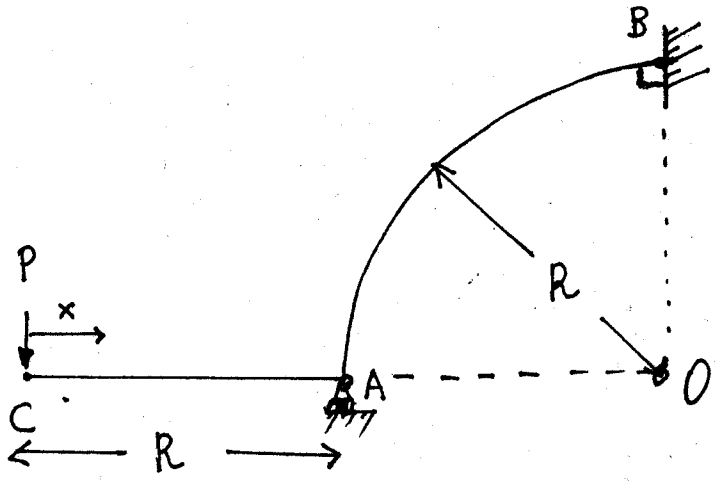


Figure 2

3. **(25 pts)** You are given the beam-column AB in Figure 3 which is subject to a horizontal compressive force P at point B. The beam is made up of two beams AC and CB of bending rigidity E_1I_1 and E_2I_2, and length L_1 and L_2 respectively which are welded at point C. The beam AC is fixed (clamped to a vertical wall) at point A and the beam CB is simply supported at point B at a frictionless pin so that the vertical displacement of point B is zero. Determine an (approximate) analytical expression for the critical buckling load P_{cr}. You may use an energy method or the differential equation method (whichever you like).

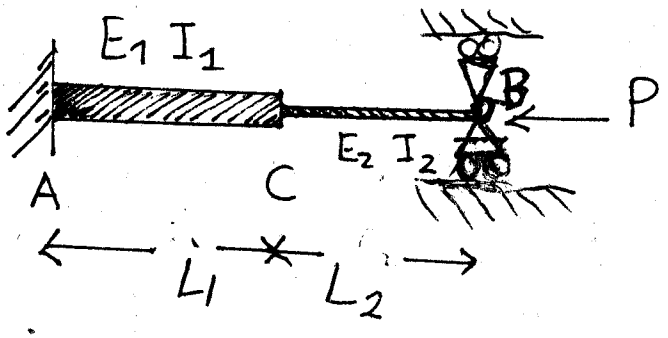


Figure 3

4. (**25 pts**) Consider a shaft of length $L = 1m$ and circular cross-section with radius $R = 8cm$ made of a linearly elastic perfectly plastic material with shear modulus $G = 0.12MPa$ and yield stress $\tau_\delta = 2kPa$ subject to a torque M_x.

   (a) (**6 pts**) Determine the maximum elastic torque the shaft can withstand as well as the relative rotation angle of the end cross-sections.

   (b) (**6 pts**) If the radius of the elastic core is $r = 1cm$, how much torque is carried by the elastic core and how much by the plastic ring around the core? In this case, what is the relative rotation angle of the end cross-sections?

   (c) (**6 pts**) Find the angle of rotation at the condition of plastic collapse of the shaft (when the entire cross-section becomes plastic).

   (d) (**7 pts**) If the torque is removed, find the residual stresses for each case (a), (b) and (c). What is the corresponding residual angle of relative rotation of the end cross-sections?