MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF OCEAN ENGINEERING AND CIVIL AND ENVIRONMENTAL ENGINEERING

13.10J/1.573J Structural Mechanics

Fall 2002

Quiz 1

Tuesday, October 8, 2002, 4 – 6 pm , Rm 3-370

FIRST READ ALL PROBLEMS
INDIVIDUAL EFFORT
CLOSED BOOK AND NOTES
ONE SHEET OF FORMULAS

Problem 1(30 pts)

a) (15pts) The stress tensor in MPa at a given point in a Cartesian XYZ coordinate system is:

$$\tau_{ij} = \begin{pmatrix} 100 & 120 & \tau_{xz} \\ \tau_{yx} & 50 & 0 \\ 220 & \tau_{zy} & \tau_{zz} \end{pmatrix}, \tau_{zz} \geq 0.$$

Given the strain energy per unit volume $U = 427450Pa$, the Young's modulus of elasticity and the Poisson ratio, $E = 200GPa$, $\nu = 0.3$ find the missing elements of the given matrix.

b) (7pts) A new frame X'Y'Z' is formed by rotating the reference frame XYZ. Under this new frame X'Y'Z', find the unknown components of stresses in the stress tensor (in MPa) below:

$$\tau'_{ij} = \begin{pmatrix} \tau'_{xx} & -50\sqrt{2} & \tau'_{xz} \\ \tau'_{yx} & 40 & 170\sqrt{2} \\ \tau'_{zx} & \tau'_{zy} & 40 \end{pmatrix}$$

c) (8pts) Given now the one eigenvalue of this new matrix in b), $\lambda_1 = 45.4748$ derive the other two eigenvalues

Problem 2 (25 pts)

Let us consider a flat plate under plane stress. Assume that the Poisson ratio is bounded: $\nu \in [0,0.5)$. Given $\varepsilon_{xx}, \varepsilon_{yy}$ show that:

a) (8pts) $\varepsilon_{zz} \leq |\varepsilon_{xx} + \varepsilon_{yy}|$

b) (17pts) Let us denote ε_1 and ε_2 the principal stresses in the plane and $\nu = 0.3$. Then if $\varepsilon = \min(\varepsilon_1, \varepsilon_2)$ show that:

i) $|\varepsilon| \leq |\varepsilon_{zz}| \frac{7}{6}$.

ii) Let us now denote $e = \max(\varepsilon_1, \varepsilon_2)$. Then show that e is bounded by:

$e \leq |\varepsilon_{zz}| \frac{7}{6} + \frac{1.3}{E}\sqrt{(A/2)^2 - B}$, where A and B are the invariants of the two dimensional stress tensor.

Problem 3 (15 pts)

a)(10 pts) The cylindrical rod given in Figure 1 has two coaxial cylindrical volumes cut out of it. It has a specific weight of γ. If the material is linear elastic with modulus of elasticity E, what is the vertical downward deflection of the top end A as a result of gravity? Give your answer in terms of $l_1, l_2, \gamma, E, d, d_a, d_b$.

b) (5pts) Before deflection occurs, due to engineering constraints $l_1 + l_2 = L$, where L is given and fixed. Determine the optimal $\frac{l_1}{l_2}$ ratio, such that deflection in question a) is minimised.

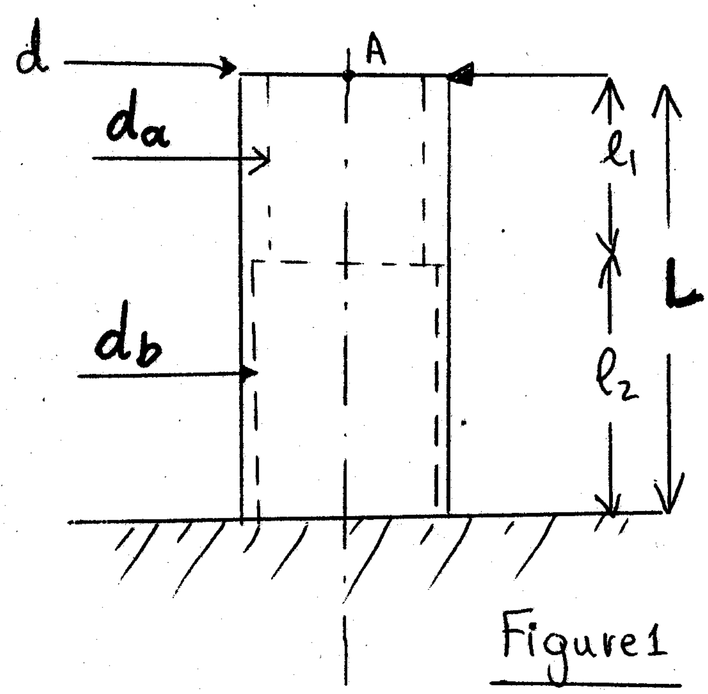

Figure1

Problem 4 (30pts)

A long rod hanging vertically in a well supports a load P at its lower end as shown in Figure 2. The material has the bilinear stress – strain curve shown in Figure 3, in which $E_1 = 2E_2$. A temperature increase ΔT is imposed. Given the load P, find ΔT such that the total elongation does not exceed δ_{tot}.

Given: α, expansion coefficient

A, cross section area

γ, specific weight

L, total length of the rod

σ^c, in Figure 3,

δ_{tot} = maximal elongation allowed.

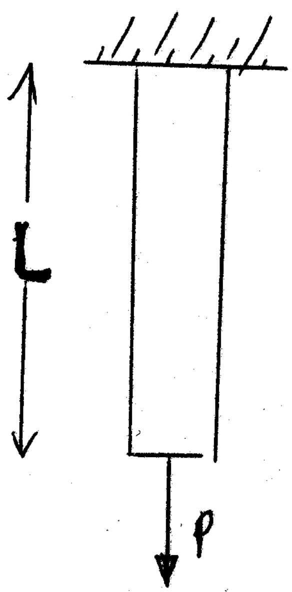

Figure 2

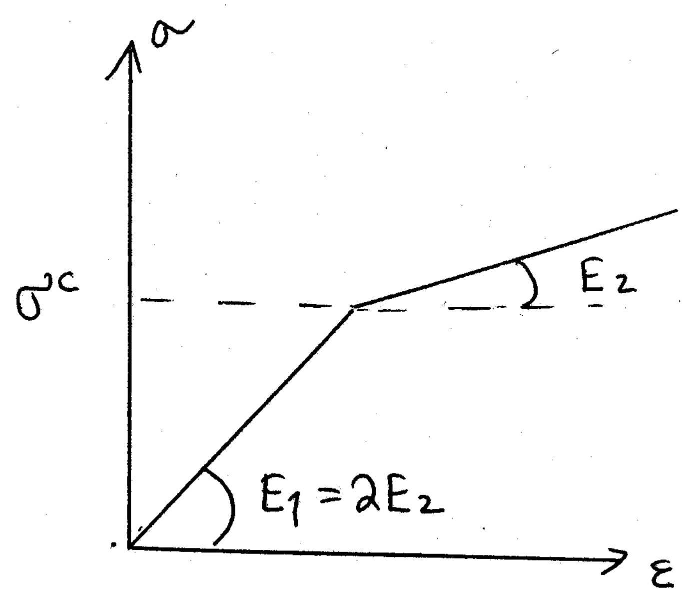

Figure3