Massachusetts Institute of Technology

Department of Ocean Engineering

Department of Civil and Environmental Engineering

Structural Mechanics 13.10J/1.573J

Quiz 2

Fall 2002

PLEASE RETURN THIS EXAM WITH YOUR SOLUTIONS

Friday, November 15, 2002, 2-4 pm

Problems

1. **Problem 1 (35 pts)**

   You are given the thin-wall cross-section in Figure 1 with thickness $t = 0.5cm$. The cross-section is subject to a vertical shear force Q parallel to axis z at the shear center.

   a) Determine the geometric centroid G of the cross-section, the moments of inertia of the cross-section about G_y, G_z and the product of inertia of the cross-section. (5 pts)

   b) Calculate the shear stress τ_{xy} at points C and D and plot the distribution of shear stress τ_{xy} along CD. (10 pts)

   c) What is the total force due to the distribution of the shear stresses on CD and TD? (8 pts)

   d) Determine the shear center of this cross-section. (7 pts)

   e) Plot the distribution of the shear stresses τ_{xy} on this cross-section. (5pts)

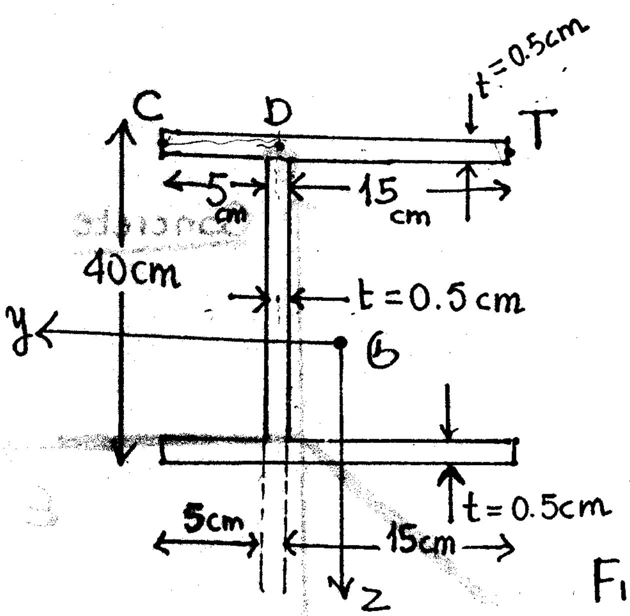


Figure 1

Problem 2 (35 pts)

You are given the following beam cross-section in Figure 2 made from concrete and two steel bars, each with cross-sectional area $A = 5cm^2$. The maximum permissible stresses in steel and concrete are given $\sigma^s = 115MPa$ and $\sigma^c = 6.5MPa$ and the ratio of the corresponding Young's modulii is $\frac{E_s}{E_c} = 20$ (for concrete in compression).

a) Plot the bending stress distribution on the cross section, due to a bending moment M_y . (10 pts)

b) Calculate the position of the neutral axis given it is between the two bars. (10 pts)

c) Determine the maximum permissible moment for this cross-section. (10 pts)

d) Describe qualitatively what happens when the above permissible stresses are exceeded. (5 pts)

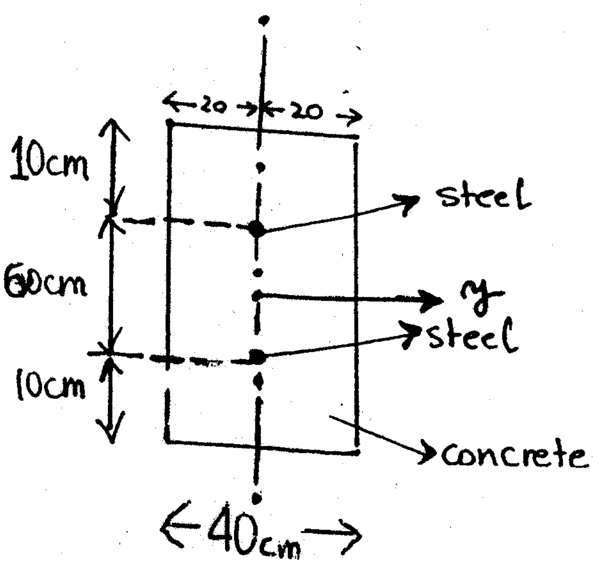
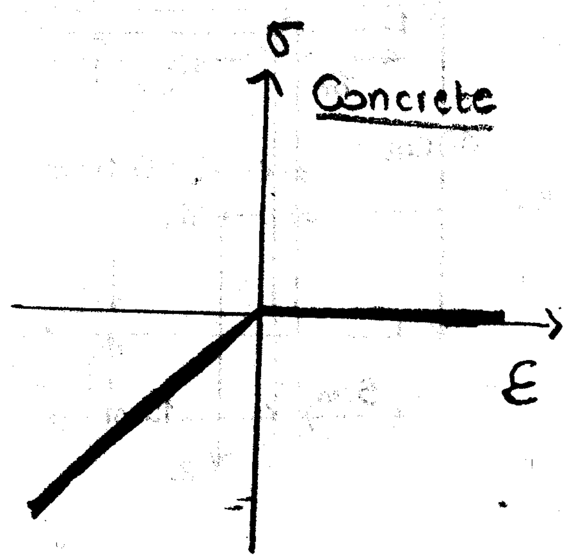


Figure 2



Problem 3 (30 pts)

You are given the following military bridge in Figure 3 supported by a cylindrical pontoon in water with density ρ_w. The pontoon's waterplane area is A_w and the acceleration of gravity is g. A military vehicle crosses the bridge with constant velocity. We model the weight of the vehicle as in Figure 4. Idealize the effect of the pontoon as an elastic spring attached to the middle of the bridge.

a) Calculate the deflection of the bridge as a function of the position of the vehicle x_v from the left end. (15 pts)

b) Determine the position x_m of the vehicle for which the maximum moment occurs and determine the corresponding point A, where such maximum bending moment occurs, as well as its value M_A. For the cross section of the bridge given in Figure 5, derive the required waterplane area A_{min}, so that the maximum bending stress does not exceed a given value σ_{max}. (15 pts)

HINT: You may use the following Figure 12.11 from the textbook.

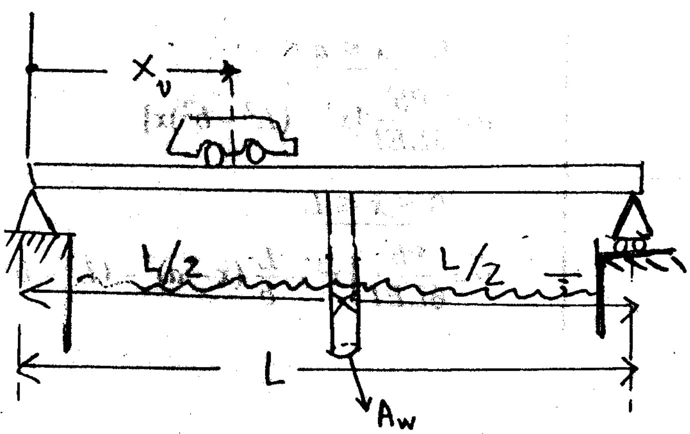


Figure 3

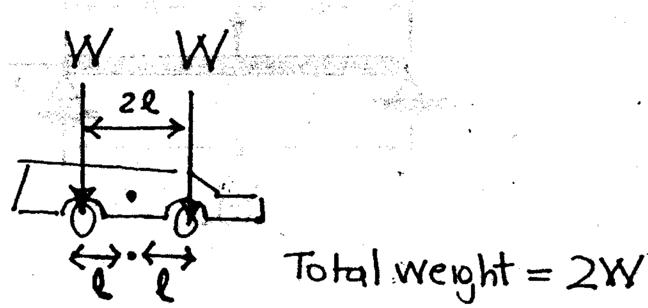


Figure 4

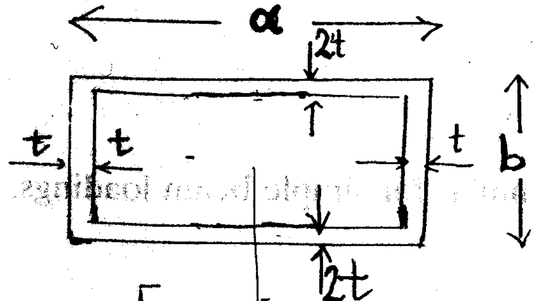
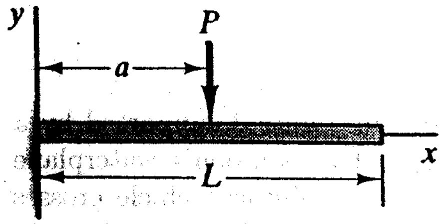


Figure 5



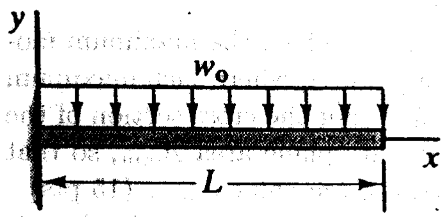
(a)

$$0 \leqslant x \leqslant 0$$

$$v = \frac{P}{6EI}\left(x^3 - 3x^2 a\right)$$

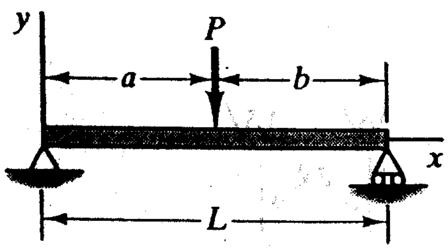
$$a \leqslant x \leqslant L$$

$$v = \frac{P}{6EI}\left[-(x-a)^3 + x^3 - 3x^2 a\right]$$



(b)

$$v = \frac{w_0 x^2}{24EI}\left(-x^2 - 6L^2 + 4Lx\right)$$



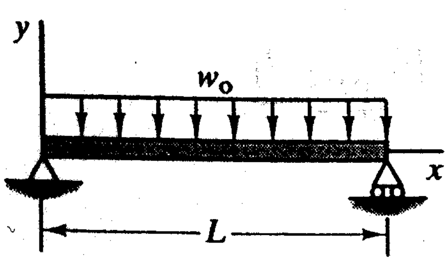
(c)

$$0 \leqslant x \leqslant a$$

$$v = \frac{Pb}{6LEI}\left[x^3 - (L^2 - b^2)x\right]$$

$$0 \leqslant x \leqslant L$$

$$v = \frac{Pb}{6LEI}\left[x^3 - \frac{L}{b}(x-a)^3 - (L^2 - b^2)x\right]$$



(d)

$$v = \frac{w_0 x}{24EI}\left(-L^3 + 2Lx^2 - x^3\right)$$

Figure 12.11. Deflection formulas for simple beam loadings.