MASSACHUSETTS INSTITUTE OF TECHNOLOGY
13.10J/1.573J/13.014/1.052
Problem Set 13

Assign : Nov. 20, 2000 (for 13.014/1.052)
Nov. 20, 2000 (for 13.10J/1.573J)
Due : Dec. 5, 2000 (for 13.014/1.052)
Dec. 6, 2000 (for 13.10J/1.573J)

For undergraduates : 18.37 and Problems 1, 2, 3 and 4 below.

For graduates : 18.37 and Problems 1, 4, 5, 6 and 7 below.

1. A slender curved circular cross section bar AB with radius of cross section r is welded to the straight cylindrical bar BC with the same radius of cross section as in Figure 1. Young's modulus of elasticity is E for both bars. AB is a quarter of the circle with radius of R. The free end is subjected to a horizontal force P. Determine the horizontal and vertical deflections δ_h, δ_v and the angle of rotation θ at the free end. Use energy method.

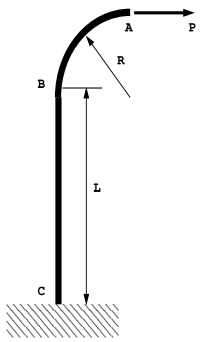
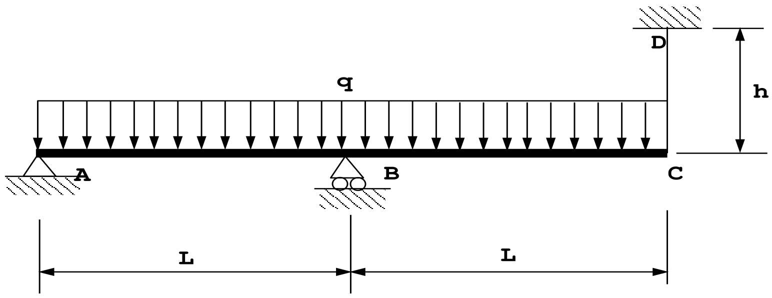


Figure 1: A diagram for Problem 1

2. A beam ABC (Figure 2) is simply supported at A and B and is hung from a cable CD at point C. Prior to the application of the uniform load q, there is no force in the cable nor is there any slack in the cable. When the load q is applied, the beam deflects downward at C and a tensile force T develops in the cable. Assume the bending rigidity of the beam is EI; the Young's modulus of the cable material is E; and the cross sectional area of the cable is A. Using the energy method, show that the magnitude of the tensile force T in the cable is

$$T = \frac{3qAL^4}{8AL^3 + 12hI} \tag{1}$$

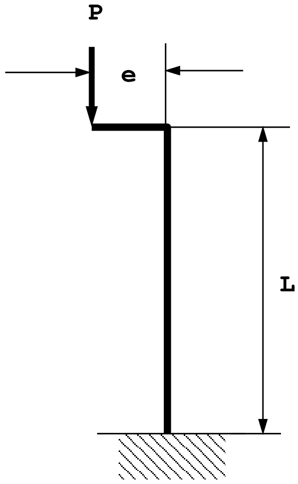


A diagram for Problem 2

3. The column shown in Figure 3 is fixed at the base and free at the upper end. A compressive load P acts at the top of the column with an eccentricity e from the axis of the column. The bending rigidity is EI. Beginning with the differential equation, show the followings:

   (a) the maximum tip deflection of the column $\delta = e(sec(kL) - 1) \quad where \;\; k = \sqrt{\frac{P}{EI}}$.

   (b) the maximum bending moment in the column $M_{max} = Pesec(kL)$.



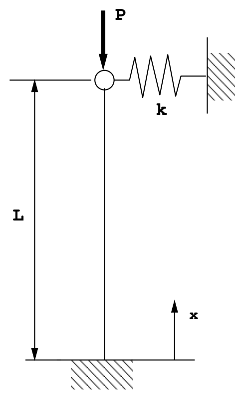
A diagram for Problem 3

4. Given the bending rigidity EI of a column of length L and the spring constant k as shown in Figure 4,

   (a) derive an analytical formula for determining the critical buckling loads of the column-spring system and show that it satisfies the following relation:

$$k\sqrt{\frac{P}{EI}}L\cos\left(\sqrt{\frac{P}{EI}}L\right) - k\sin\left(\sqrt{\frac{P}{EI}}L\right) + P\sqrt{\frac{P}{EI}} = 0 \tag{2}$$

(b) Using the energy method, find an approximate solution to estimate the first critical buckling load P_{cr}. Assume that the deflection curve is $y(x) = ax^2$ and show that the first critical buckling load is $P_{cr} = \frac{3EI}{L^2} + \frac{3kL}{4}$.



A diagram for Problem 4

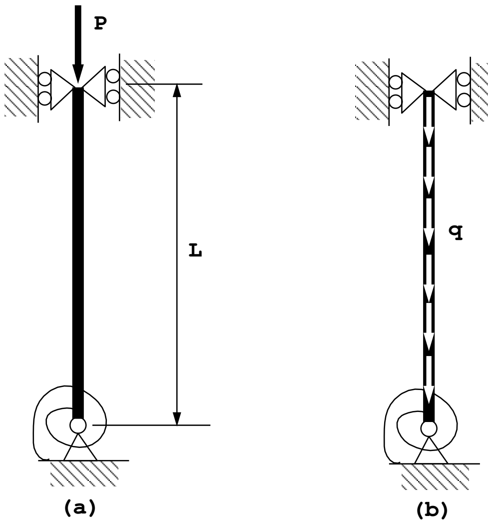
5. A column of length L and bending rigidity EI is pin-connected at both ends and at the same time is connected at the lower end to a torsional spring (resisting rotation) having a spring constant of K_T as shown in Figure 5.

(a) When the column is subjected to axial load P, show that the equation for critical load P_{cr} for buckling is

$$tan(kL) = \frac{kL}{1 + \frac{EI}{K_T}k^2} \quad where\ \ k = \sqrt{\frac{P}{EI}} \tag{3}$$

(b) Instead of P, the column is under the action of uniformly distributed axial force per unit length q (e.g. its weight) as shown in Figure 4-(b). Using the energy method, prove that q_{cr} is given as follows. Assume the deflection curve is $y = A\sin(\frac{\pi x}{L})$.

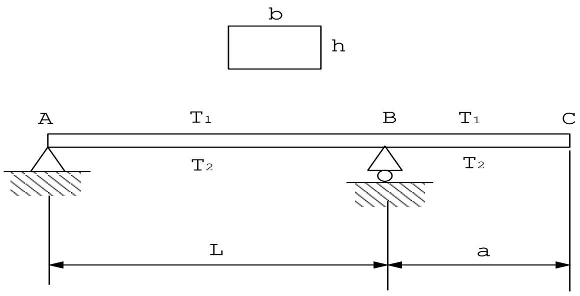
$$q_{cr} = \frac{4K_T}{L^2} + \frac{2EI\pi^2}{L^3} \tag{4}$$



A diagram for Problem 5

6. An overhanging beam ABC with rectangular cross section of height h is heated to a temperature T_1 on the top and T_2 $(> T_1)$ on the bottom (see Figure 6). The coefficient of thermal expansion of the material of the beam is α. Assume the temperature within the thickness varies linearly. Young's modulus of elasticity is E. Show that the vertical deflection δ_c at the free end C of the overhang is

$$\delta_c = \frac{a\alpha}{2h}(T_2 - T_1)(L + a) \tag{5}$$



A diagram for Problem 6

7. A *rigid* bar is supported by five columns of length L, cross sectional area A_i and Young's modulus of elasticity E_i as shown in Figure 7. Considering tension and compression of the columns only, answer the following questions. (Assuming NO BUCKLING.)

(a) Express the total strain energy as a function of v, θ and L under the action of moment M at B. Here v and θ are the vertical displacement and rotation of the rigid bar about point O, respectively.

(b) Show that the vertical displacement v and rotation angle θ at O under the action of moment M at B are

$$
\begin{aligned}
v &= \frac{-b}{ac-b^2}M \\
\theta &= \frac{a}{ac-b^2}M \\
where \quad a &= \sum_{i=1}^{5}\frac{E_iA_i}{L} \qquad b = \sum_{i=1}^{5}E_iA_i|3-i|, \\
c &= \sum_{i=1}^{5}E_iA_i(3-i)^2L
\end{aligned}
\tag{6}
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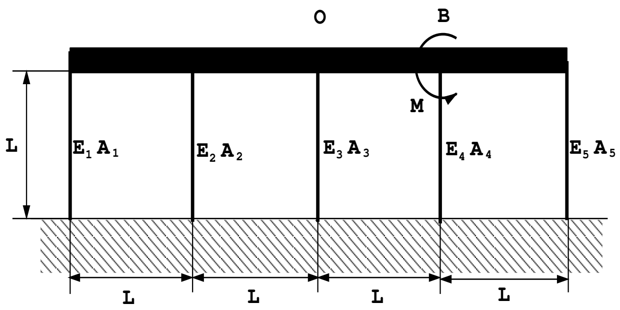

A diagram for Problem 7