MASSACHUSETTS INSTITUTE OF TECHNOLOGY
13.10J/1.573J
Problem Set 13

Assign : Nov. 27, 2002

Due : Dec. 6, 2002 (in Recitation)

For graduates : 18.37, 18.38, 18.39 and the problems beloww.

1. A column of length L and bending rigidity EI is pin-connected at both ends and at the same time is connected at the lower end to a torsional spring (resisting rotation) having a spring constant of K_T as shown in Figure 1.

   (a) When the column is subjected to axial load P, show that the equation for critical load P_{cr} for buckling is

   $$tan(kL) = \frac{kL}{1 + \frac{EI}{K_T}k^2} \quad where\ k = \sqrt{\frac{P}{EI}} \tag{1}$$

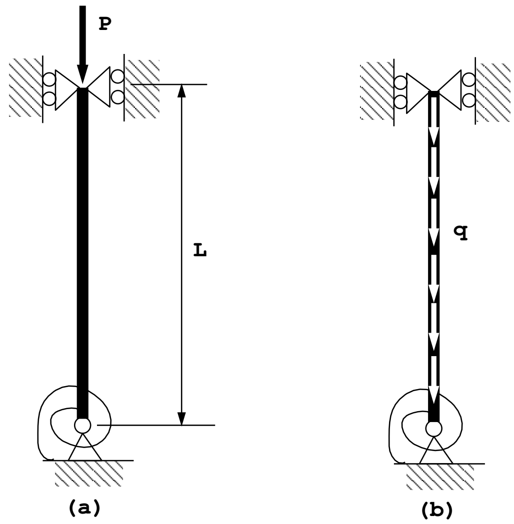
   (b) Instead of P, the column is under the action of uniformly distributed axial force per unit length q (e.g. its weight) as shown in Figure 4-(b). Using the energy method, prove that q_{cr} is given as follows. Assume the deflection curve is $y = A\sin(\frac{\pi x}{L})$.

   $$q_{cr} = \frac{4K_T}{L^2} + \frac{2EI\pi^2}{L^3} \tag{2}$$

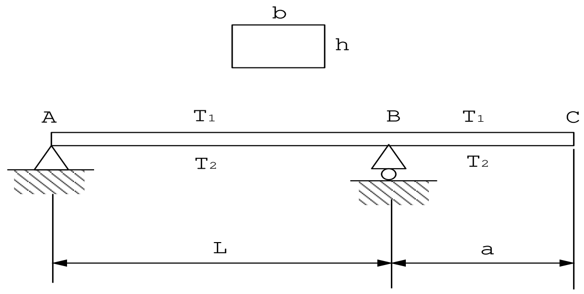
2. An overhanging beam ABC with rectangular cross section of height h is heated to a temperature T_1 on the top and T_2 $(> T_1)$ on the bottom (see Figure 2). The coefficient of thermal expansion of the material of the beam is α. Assume the temperature within the thickness varies linearly. Young's modulus of elasticity is E. Show that the vertical deflection δ_c at the free end C of the overhang is

   $$\delta_c = \frac{a\alpha}{2h}(T_2 - T_1)(L + a) \tag{3}$$

3. A *rigid* bar is supported by five columns of length L, cross sectional area A_i and Young's modulus of elasticity E_i as shown in Figure 2. Considering tension and compression of the columns only, answer the following questions. (Assuming NO BUCKLING.)



A diagram for Problem 1



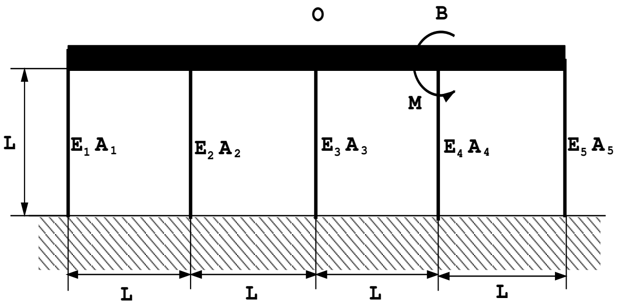
A diagram for Problem 2

(a) Express the total strain energy as a function of v, θ and L under the action of moment M at B. Here v and θ are the vertical displacement and rotation of the rigid bar about point O, respectively.

(b) Show that the vertical displacement v and rotation angle θ at O under the action of moment M at B are

$$\begin{aligned} v &= \frac{-b}{ac - b^2} M \\ \theta &= \frac{a}{ac - b^2} M \\ where \quad a &= \sum_{i=1}^{5} \frac{E_i A_i}{L} \quad b = \sum_{i=1}^{5} E_i A_i |3 - i|, \end{aligned} \tag{4}$$

$$c = \sum_{i=1}^{5} E_i A_i (3-i)^2 L$$



A diagram for Problem 3