MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF OCEAN ENGINEERING AND CIVIL AND ENVIRONMENTAL ENGINEERING
13.10J/1.573J Introduction to Structural Mechanics
Fall 1999
Problem Set 9

Assigned : November 4, 1999
Due : Part I - November 16, 1999
Due : Part II - November 23, 1999

Part I

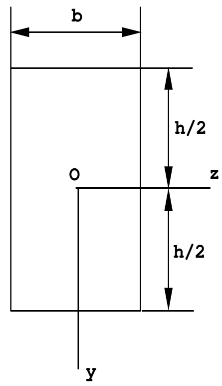
1. A problem from Chapter 11 of the course textbook.

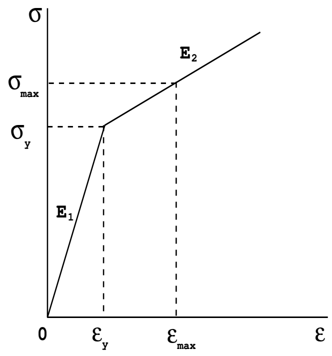
   (a) 11.74

2. Derive the following moment-curvature equation for a beam of rectangular cross section having a bilinear stress-strain diagram. The compression part of the $\sigma - \epsilon$ diagram is anti-symmetric.

$$\frac{M}{M_y} = \frac{1}{2}\left[\left(1 - \frac{E_2}{E_1}\right)\left(3 - \frac{\kappa_y^2}{\kappa^2}\right) + \frac{2E_2\kappa}{E_1\kappa_y}\right] \quad \left(\frac{M}{M_y} \geq 1\right) \tag{1}$$

   in which $M_y = \sigma_y S = \sigma_y bh^2/6$ and $\kappa_y = 2\epsilon_y/h = 2\sigma_y/E_1 h$.

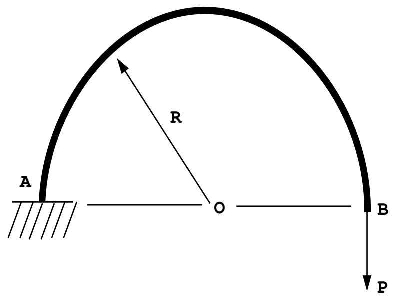




3. A slender curved circular cross-section bar AB with radius of cross-section r and Young's modulus of elasticity, E, which is of semicircular shape with radius R is fixed at support A and free at B. A vertical load P acts at the free end.

   (a) Determine the horizontal and vertical deflections δ_h, δ_v, and the angle of rotation θ at the free end.

   (b) Find the radius r of the cross section of the curved bar if the allowable maximum vertical deflection at B is δ_o.



4. Problems from Chapter 14 of the course textbook.

   (a) 14.18

   (b) 14.34

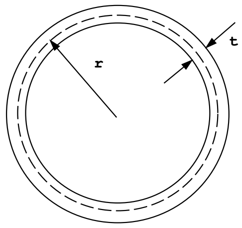
   (c) 14.42

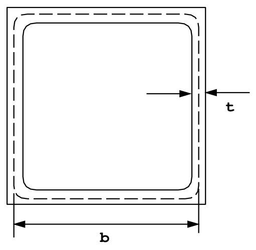
   (d) 14.46

5. A circular tube and a square tube are constructed of the same material with yield stress τ_y and shear modulus of elasticity G. Both tubes have the same length, thickness, and material cross-sectional area, and both are subjected to the same torque.

   (a) What are the ratios of the shear stresses and of the angles of twist for the tubes? (Disregard the effects of stress concentrations at the corners of the square tube.)

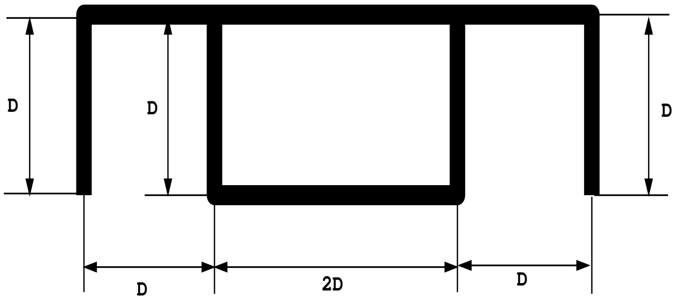
   (b) Find t and b for the square tube which minimize the weight of the tube under the conditions that $t \geq t_0$ and safety factor n with respect to yielding stress in shear, τ_y when an external torque T is applied.





Part II

6. The following figure shows the cross section of a bridge-like beam. Calculate the torsional rigidity and the rate of twist for a torque T. All sections have thickness t, and $t << D$. The shear modulus of the material is G.



7. A thick-walled steel cylinder with open ends is subjected to an inside pressure p. The outer pressure is assumed to be zero.

   (a) Find the radial displacement $u_r(r)$. First show that $u_r(r)$ obeys the ODE

   $$r^2 \frac{\partial^2 u_r}{\partial r^2} + r \frac{\partial u_r}{\partial r} - u_r = 0.$$

   The general solution of this equation is of the form $u_r(r) = r^m$.)

   (b) Prove that the radial and circumferential stresses are given by

   $$\begin{aligned} \sigma_{rr}(r) &= \frac{pb^2}{b^2 - a^2}\left(\frac{a^2}{r^2} - 1\right), \\ \sigma_{\theta\theta}(r) &= \frac{pb^2}{b^2 - a^2}\left(\frac{a^2}{r^2} + 1\right). \end{aligned}$$

   (c) Find the location and magnitude of the maximum normal and shear stress

   (d) Find the pressure corresponding to first yield. (Use von Mises and Tresca yield condition.)

8. A rigid-perfectly plastic beam of the length l and cross-section $a \times b$ is subjected to a uniform load q. The beam is fully clamped at one end and free at the other.

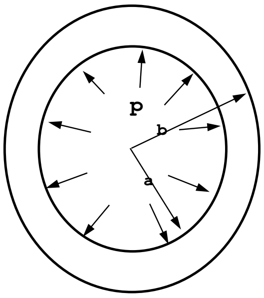


Figure 1: Problem 7

(a) Determine the load-carrying capacity of the beam, q_c (the maximum external load that the beam can resist), as a function of a, b, l and the yield stress of the material σ_y.

(b) How this critical load compares to the "first" yield load, q_y? The first yield is reached where mostly stressed point in the beam reaches the yield stress.
(Hint: Solve the elastic problem for the beam with bending rigidity EI.)

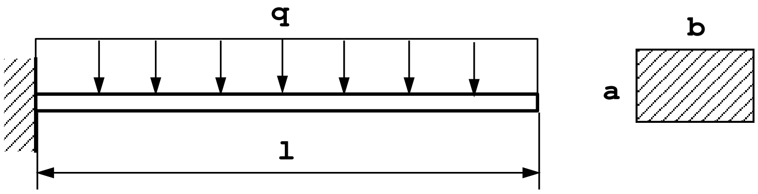


Figure 2: Problem 8

Problem sets are available also in the course homepage.