MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENTS OF OCEAN, MECHANICAL,
CIVIL & ENVIRONMENTAL AND AERONAUTICAL &
ASTRONAUTICAL ENGINEERING
13.472J/2.158J/1.128J/16.940J/Computational Geometry
Spring Term, 2000

Problem Set #3 on Blends, GCs and intersections
Issued: Thursday, March 14, 2000
Due: Tuesday, March 28, 2000
Weight: 15% of total grade

Problem 1: Blending Surfaces

An active area of research is in creating *blending surfaces* — smooth joins between two objects. As an example we will consider developing a blending surface between the plane $z = 0$ and the right circular cylinder $x^2 + y^2 = 1,\ 0 \leq z \leq 2$. Because the cylinder is a surface of revolution, we will, for simplicity, consider the cross-section of the objects obtained by setting $y = 0$ as shown in the figure below. Our problem is to develop a smooth surface between the cylinder and the plane by creating a cross-section curve starting at height 1 on the cylinder and terminating on the plane 2 units away from the origin.

a. As a first effort, consider using a quadratic Bezier curve (i.e. a parabola) as a blend cross-section. The curve should have the starting and ending points indicated on the diagram, and to ensure a smooth blend, the tangent to the curve at the start (end) point should have the same direction as the tangent to the cylinder (plane). Give the control points of this curve.

b. Using the results of part **a**, express the blending surface (i.e. the surface of revolution characterized by the cross section obtained in **a**) as rational B-spline (NURBS) surface.

c. Now suppose we want a cubic Bezier curve as the cross-section of our blending surface. Give the control points of a cubic Bezier curve which generates a "good" blend. To be "good", the curve not only has to satisfy the boundary conditions indicated in part **a**, but also the area under the curve (the shaded area in the figure) should be between 0.2 and 0.3.

d. Now suppose we want to maintain curvature countinuity at the blending surface linkage curves in addition to position and tangent plane continuity. Determine a sufficiently high degree Bezier curve cross section to accomplish this.

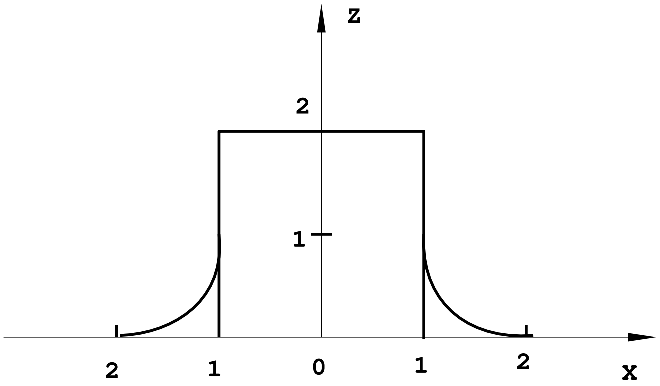

Figure 1: Blends

Problem 2: Intersection of an Algebraic surface with a Parametric Curve

a. Give the implicit polynomial equation of a torus whose section circle has radius 2, and whose center circle has radius 4. Assume the torus is situated so that it is centered at the origin and the center circle lies entirely in the (x, y)-plane.

b. Using the implicit equation in **a**, compute all intersections of the torus with the cubic Bézier curve having control points $\mathbf{r}_0 = (0, 6, 0)$, $\mathbf{r}_1 = (-5, 2, -0.5)$, $\mathbf{r}_2 = (2, -3, 0.5)$, $\mathbf{r}_3 = (6, 0, 0)$. Give both the Cartesian coordinates of the intersection points and their associated parameter values on the Bézier curve. Indicate which method you used to solve this problem, and give all answers to at least 5 significant digits.

(*You may use Maple or another symbolic manipulation program to do the substitution and solution.*)

Problem 3: Planar curve intersections

Consider the following curves:

a. An implicit planar algebraic curve:

$$\begin{aligned} f(u,v) = & - \quad 64v^4 + 128v^3 - 96u^2v^2 + 140uv^2 - 139v^2 + 96u^2v - 140uv + 75v \\ & - \quad 96u^4 + 276u^3 - 313u^2 + 165u - 36 = 0 \end{aligned}$$

and

b. A (parametric) planar Bézier curve (in the same plane as the above implicit curve)

$$\mathbf{r}(t) = [u(t), v(t)] = \sum_{i=0}^{3} \mathbf{r}_i B_{i,3}(t)$$

where $B_{i,3}$ denotes the ith cubic Bernstein polynomial and $\mathbf{r}_0 = (0.5, 0.5)$, $\mathbf{r}_1 = (0.7, 0.6)$, $\mathbf{r}_2 = (0.95, 0.1)$, $\mathbf{r}_3 = (0.55, 0.25)$.

a. Compute all turning and singular points of $f(u, v)$ to the highest possible accuracy, as well as the tangent lines at all these points.

b. Using the results of **a** as a guide, sketch $f(u, v)$. Clearly indicate the turning and singular points on your sketch.

c. Compute the intersections of the two curves given above to the highest possible accuracy. In addition to giving the Cartesian coordinates of the intersection points, also include the parameter values of the points on the Bézier curve and their multiplicity.

(*You may use Maple or another symbolic manipulation program to do the substitution and solution.*)