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Similarity Detection Algorithms for CAD Models

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Abstract: This document presents two spectral methods for shape matching and recognition. Our first method computes the Laplace-Beltrami spectra on a domain to describe its shape. Since the spectrum is an isometry invariant and contains geometrical information, it is optimally suited for shape analysis and shape matching of geometric data. We have recently focused on global shape comparison with an application to medical imaging. In our second approach, a wavelet-based spectral method has been studied. A new form of wavelet transform is employed to greatly reduce the sensitivity to translation and rotation of a model. We also take advantage of the multi-resolution attribute to operate on reduced data sets at lower resolutions, where we are able to rapidly detect the similarity

1. Laplace Spectra for Shape Recognition: This section describes our study on global shape analysis based on the Laplace-Beltrami spectrum of a Riemannian manifold [1,2,3,4,5] with an application in the field of medical imaging [6,7]. Previous approaches for global shape analysis in medical imaging include the use of invariant moments [8], the shape index [9], and global shape descriptors based on spherical harmonics [10]. Our methodology based on the Laplace-Beltrami spectrum differs in the following ways from these previous approaches:

- It works for any Riemannian manifold, whereas spherical harmonics based methods are restricted to surfaces with spherical topology, and invariant moments do not easily generalize to arbitrary Riemannian manifolds. It may thus be used to analyze surface, solids, non-spherical objects, etc.
- In the 2D case the only preprocessing requirement is the extraction of a surface approximation from the manually segmented binary volume. In 3D the method runs directly on the binary volume representation (voxel) without any preprocessing. No registration, remeshing, or additional mappings are necessary. Furthermore, the description is invariant to translations, rotations, isometries, and surface meshing.

Given the Laplace-Beltrami operator Δ of a real-valued function f, with $f \in C^2$, defined on a Riemannian manifold $M : \Delta f := div(grad\ f)$ then the Helmholtz equation (also known as the Laplacian eigenvalue problem) is stated as $\Delta f = -\lambda f$. If M is bounded we employ either the Dirichlet $(f \equiv 0)$ or the Neumann boundary condition $(\partial f / \partial n \equiv 0)$ on the boundary. The solutions of this equation represent the spatial part of the solutions of the wave equation, with an infinite number of eigenvalue λ and eigenfunction f pairs. The possibly normalized beginning sequence of the Dirichlet or Neumann spectrum (the first n eigenvalues – called ShapeDNA) can be used as a fingerprint of the object's intrinsic geometry (independent of the dimension of M – e.g. 2D surface or 3D solid). The ShapeDNA can be successfully applied for database retrieval. See Figure 1 for a plot presenting the first two principal components of the high dimensional ShapeDNA of a few closed surfaces. It can be seen how the ShapeDNA clusters the objects into meaningful groups (considering that the helmet is in fact a deformed ellipsoid where one cap has been flipped to the inside).

Recently the ShapeDNA was applied to medical data representing two populations of brain parts (caudate nucleus obtained from MRI scans, see Figure 2) of female subjects diagnosed with Schizotypal Personality Disorder (SPD) and normal control (NC) subjects [6,7]. It could be demonstrated that the ShapeDNA can pick up statistically significant differences of the two populations not only in size, but also in shape, confirming the influence of SPD on the brain part (caudate nucleus). Furthermore, it could be shown that computing the Dirichlet and Neumann spectra directly on the 3D voxel data is already feasible on a standard PC. The Neumann spectra are of interest, since they recognize shape differences much earlier than the Dirichlet spectra and also work much better if the voxel resolution is very low. Especially the higher eigenvalues yielded statistically significant results, indicating that true shape differences exist mainly in areas with smaller features (e.g. the tail area of the caudates). This result is very promising since it demonstrates the possibility of comparing shapes based on a specific size of the features of interest (multiresolutional shape matching).

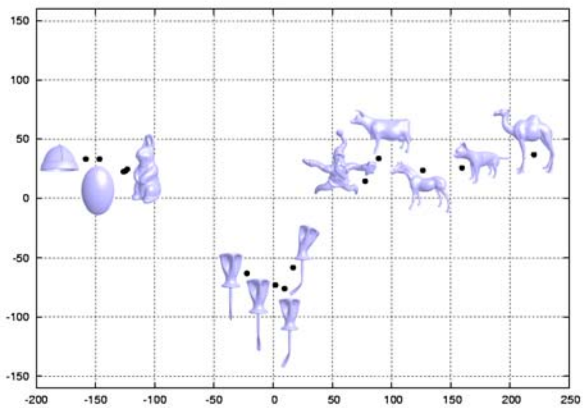

Figure 1: Principal components showing the position of the high-dimensional shapeDNA for the corresponding objects

Figure 2: Example of a caudate (brain part)

2. Shape Matching of 3D Objects Using the Wavelet Transform: Recent developments in shape-based modeling and data acquisition have brought three-dimensional models to the forefront of computational research. Three-dimensional shape-based matching is a key in such activities as inspection, computer-enhanced surgery, and computer vision. Many of the 3D models consist of huge datasets; a reduction in the amount of data which can be achieved through a transform to an auxiliary space can speed the matching. Features in wavelet space can provide appropriate landmarks for registration (alignment) and for similarity evaluation. They also provide a multi-scale adaptive approach progressing from low resolution to high resolution, thus allowing initial coarse matching at lower resolutions.

A major drawback to the wavelet transform is its lack of invariance to rotation or translation. A newly developed transform, called the dual-tree complex wavelet transform (DTCWT) [11] is nearly invariant to translation and much less sensitive to rotation than the traditional discrete wavelet transform. We employ the DTCWT to accomplish registration and matching of three-dimensional voxelized objects by extracting features in a multi-resolutional format. In this work [12], we include noise due to transformation, sensing error, voxelization, etc., but do not include shearing, non-uniform scaling, warping, or other motions that distort the appearance or functionality of an object.

The algorithm is as follows:

1. Apply the wavelet transform to both objects and select the level of resolution at which matching is performed. The three-dimensional DTCWT produces 28 complex sub-bands of wavelet coefficient information; instead of using 28 separate data sets, we reconstruct one level of wavelet coefficients, excluding the average coefficients, in order to produce a single set of wavelet coefficient data. These detail coefficients provide clear features for use in matching.
2. Select the extrema of the reconstructed wavelet coefficients as feature points. Figure 3 is a two-dimensional example of feature point selection; note that obvious points such as the intersections of lines are automatically selected as features. Although the DTCWT is nearly shift invariant and much less affected by rotation than other discrete wavelet methods, it is not completely invariant to rotation and translation, producing a set of feature points with many outliers.
3. We obtain potentially corresponding triplets of points in models A and B, by selecting those with the same distances between the three points, within some tolerance. For these potentially matching triplets, we calculate a transformation matrix, then use a voting scheme to select the proper transformation.
4. The quality of the match is a measure based on the Hausdorff distance. If the registration is insufficiently close, we can refine the registration using a method such as iterated closest point.

We have conducted global matching of many example models including both continuously changing models such as MRI scans, and surface models which have been voxelized with a constant value in the interior. One example is an MRI scan of a brain; see Figure 4.
We are currently researching the application of this method to partial matching, including the accommodation of occlusion and clutter.

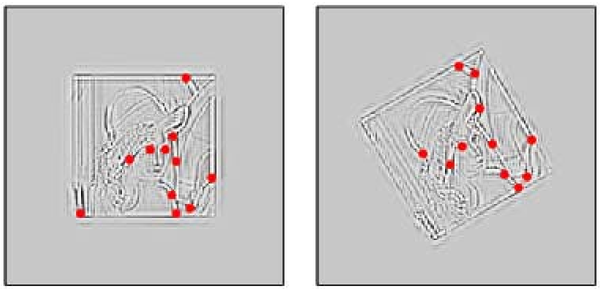

Figure 3: A two dimensional example of extrema in the DTCWT. The left figure shows the level 3 reconstructed wavelet coefficients of the original image. The right figure shows the level 3 reconstructed wavelet coefficients of the rotated/translated image. Extrema are denoted by red dots.

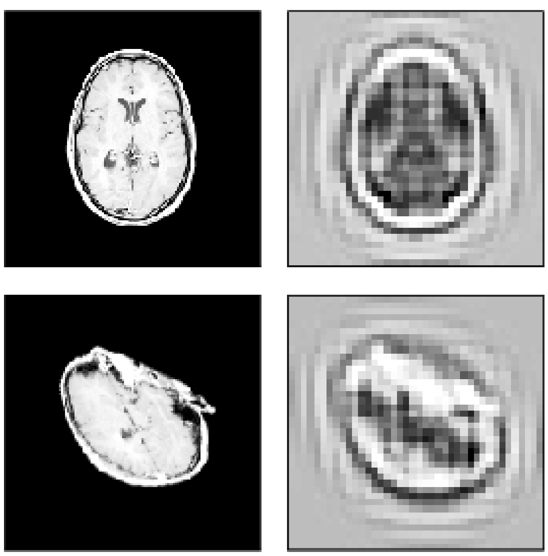

Figure 4: MRI brain scan. The top images: one slice from the unrotated object. The bottom images: rotated 10, 20, and 30 degrees in a pitch-roll-yaw Euler angle scheme. The left images: original object; the right images: the reconstructed third level wavelet coefficients. The original object is a 160×160×160 cube. At the fourth level DTCWT with reconstructed wavelets, the object is a 40×40×40 cube. The registration algorithm performed with 20 feature points in each object produces the correct rotation/translation matrix.

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