

Contents

1. Representation of Curves and Surfaces	1
1.1 Analytic representation of curves	1
1.1.1 Plane curves	1
1.1.2 Space curves	3
1.2 Analytic representation of surfaces	4
1.3 Bézier curves and surfaces	6
1.3.1 Bernstein polynomials	6
1.3.2 Arithmetic operations of polynomials in Bernstein form	7
1.3.3 Numerical condition of polynomials in Bernstein form .	9
1.3.4 Definition of Bézier curve and its properties	12
1.3.5 Algorithms for Bézier curves	13
1.3.6 Bézier surfaces	18
1.4 B-spline curves and surfaces	20
1.4.1 B-splines	20
1.4.2 B-spline curve	21
1.4.3 Algorithms for B-spline curves	24
1.4.4 B-spline surface	29
1.5 Generalization of B-spline to NURBS	30
2. Differential Geometry of Curves	35
2.1 Arc length and tangent vector	35
2.2 Principal normal and curvature	39
2.3 Binormal vector and torsion	43
2.4 Frenet-Serret formulae	47
3. Differential Geometry of Surfaces	49
3.1 Tangent plane and surface normal	49
3.2 First fundamental form I (metric)	52
3.3 Second fundamental form II (curvature)	55
3.4 Principal curvatures	59
3.5 Gaussian and mean curvatures	64
3.5.1 Explicit surfaces	64
3.5.2 Implicit surfaces	65
3.6 Euler's theorem and Dupin's indicatrix	68

4. Nonlinear Polynomial Solvers and Robustness Issues	73
4.1 Introduction	73
4.2 Local solution methods	74
4.3 Classification of global solution methods	76
4.3.1 Algebraic and Hybrid Techniques	76
4.3.2 Homotopy (Continuation) Methods	78
4.3.3 Subdivision Methods	78
4.4 Projected Polyhedron algorithm	78
4.5 Auxiliary variable method for nonlinear systems with square roots of polynomials	88
4.6 Robustness issues	90
4.7 Interval arithmetic	92
4.8 Rounded interval arithmetic and its implementation	95
4.8.1 Double precision floating point arithmetic	95
4.8.2 Extracting the exponent from the binary representation	98
4.8.3 Comparison of two different <i>unit-in-the-last-place</i> implementations	101
4.8.4 Hardware rounding for rounded interval arithmetic	102
4.8.5 Implementation of rounded interval arithmetic	103
4.9 Interval Projected Polyhedron algorithm	105
4.9.1 Formulation of the governing polynomial equations	105
4.9.2 Comparison of software and hardware rounding	106
5. Intersection Problems	109
5.1 Overview of intersection problems	109
5.2 Intersection problem classification	111
5.2.1 Classification by dimension	112
5.2.2 Classification by type of geometry	113
5.2.3 Classification by number system	114
5.3 Point/point intersection	114
5.4 Point/curve intersection	115
5.4.1 Point/implicit algebraic curve intersection	115
5.4.2 Point/rational polynomial parametric curve intersection	117
5.4.3 Point/procedural parametric curve intersection	121
5.5 Point/surface intersection	121
5.5.1 Point/implicit algebraic surface intersection	121
5.5.2 Point/rational polynomial parametric surface intersection	122
5.5.3 Point/procedural parametric surface intersection	125
5.6 Curve/curve intersection	126
5.6.1 Rational polynomial parametric/implicit algebraic curve intersection (Case D3)	126
5.6.2 Rational polynomial parametric/rational polynomial parametric curve intersection (Case D1)	130

5.6.3	Rational polynomial parametric/procedural parametric and procedural parametric/procedural parametric curve intersections (Cases D2 and D5)	131
5.6.4	Procedural parametric/implicit algebraic curve intersection (Case D6)	133
5.6.5	Implicit algebraic/implicit algebraic curve intersection (Case D8)	133
5.7	Curve/surface intersection	134
5.7.1	Rational polynomial parametric curve/implicit algebraic surface intersection (Case E3)	135
5.7.2	Rational polynomial parametric curve/rational polynomial parametric surface intersection (Case E1)	135
5.7.3	Rational polynomial parametric/procedural parametric and procedural parametric/procedural parametric curve/surface intersections (Cases E2/E6)	136
5.7.4	Procedural parametric curve/implicit algebraic surface intersection (Case E7)	136
5.7.5	Implicit algebraic curve/implicit algebraic surface intersection (Case E11)	137
5.7.6	Implicit algebraic curve/rational polynomial parametric surface intersection (Case E9)	137
5.8	Surface/surface intersections	137
5.8.1	Rational polynomial parametric/implicit algebraic surface intersection (Case F3)	138
5.8.2	Rational polynomial parametric/rational polynomial parametric surface intersection (Case F1)	147
5.8.3	Implicit algebraic/implicit algebraic surface intersection (Case F8)	151
5.9	Overlapping of curves and surfaces	155
5.10	Self-intersection of curves and surfaces	157
5.11	Summary	159
6.	Differential Geometry of Intersection Curves	161
6.1	Introduction	161
6.2	More differential geometry of curves	162
6.3	Transversal intersection curve	164
6.3.1	Tangential direction	164
6.3.2	Curvature and curvature vector	165
6.3.3	Torsion and third order derivative vector	167
6.3.4	Higher order derivative vector	168
6.4	Intersection curve at tangential intersection points	170
6.4.1	Tangential direction	171
6.4.2	Curvature and curvature vector	173
6.4.3	Third and higher order derivative vector	176
6.5	Examples	177

6.5.1	Transversal intersection of parametric-implicit surfaces	177
6.5.2	Tangential intersection of implicit-implicit surfaces . . .	179
7.	Distance Functions	181
7.1	Introduction	181
7.2	Problem formulation	182
7.2.1	Definition of the distances between two point sets	182
7.2.2	Geometric interpretation of stationarity of distance function	184
7.3	More about stationary points	185
7.3.1	Classification of stationary points	185
7.3.2	Nonisolated stationary points	190
7.4	Examples	192
8.	Curve and Surface Interrogation	195
8.1	Classification of interrogation methods	195
8.1.1	Zeroth-order interrogation methods	196
8.1.2	First-order interrogation methods	197
8.1.3	Second-order interrogation methods	200
8.1.4	Third-order interrogation methods	205
8.1.5	Fourth-order interrogation methods	208
8.2	Stationary points of curvature of free-form parametric surfaces	210
8.2.1	Gaussian curvature	210
8.2.2	Mean curvature	213
8.2.3	Principal curvatures	214
8.3	Stationary points of curvature of explicit surfaces	215
8.4	Stationary points of curvature of implicit surfaces	221
8.5	Contouring constant curvature	223
8.5.1	Contouring levels	223
8.5.2	Finding starting points	223
8.5.3	Mathematical formulation of contouring	225
8.5.4	Examples	227
9.	Umbilics and Lines of Curvature	231
9.1	Introduction	231
9.2	Lines of curvature near umbilics	232
9.3	Conversion to Monge form	237
9.4	Integration of lines of curvature	242
9.5	Local extrema of principal curvatures at umbilics	244
9.6	Perturbation of generic umbilics	250
9.7	Inflection lines of developable surfaces	256
9.7.1	Differential geometry of developable surfaces	256
9.7.2	Lines of curvature near inflection lines	262

10. Geodesics	265
10.1 Introduction	265
10.2 Geodesic equation	266
10.2.1 Parametric surfaces	266
10.2.2 Implicit surfaces	270
10.3 Two point boundary value problem	272
10.3.1 Introduction	272
10.3.2 Shooting method	273
10.3.3 Relaxation method	274
10.4 Initial approximation	275
10.4.1 Linear approximation	275
10.4.2 Circular arc approximation	277
10.5 Shortest path between a point and a curve	278
10.6 Numerical applications	281
10.6.1 Geodesic path between two points	281
10.6.2 Geodesic path between a point and a curve	282
10.7 Geodesic offsets	284
10.8 Geodesics on developable surfaces	287
11. Offset Curves and Surfaces	293
11.1 Introduction	293
11.1.1 Background and motivation	293
11.1.2 NC machining	293
11.1.3 Medial axis	299
11.1.4 Tolerance region	306
11.2 Planar offset curves	307
11.2.1 Differential geometry	307
11.2.2 Classification of singularities	308
11.2.3 Computation of singularities	311
11.2.4 Approximations	312
11.3 Offset surfaces	316
11.3.1 Differential geometry	316
11.3.2 Singularities of offset surfaces	318
11.3.3 Self-intersection of offsets of implicit quadratic surfaces	319
11.3.4 Self-intersection of offsets of explicit quadratic surfaces	328
11.3.5 Self-intersection of offsets of polynomial parametric surface patches	335
11.3.6 Tracing of self-intersection curves	343
11.3.7 Approximations	345
11.4 Pythagorean hodograph	349
11.4.1 Curves	349
11.4.2 Surfaces	351
11.5 General offsets	352
11.6 Pipe surfaces	353
11.6.1 Introduction	353

11.6.2 Local self-intersection of pipe surfaces	355
11.6.3 Global self-intersection of pipe surfaces	356
Problems	367
A. Color Plates	377
References	381
Index	405