Optimal 3D Dithering Pattern for 3DP

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November 6, 1998

Optimal Volume Halftoning

Minimization of the *longest finite wavelength* of the nonzero sinusoidal components that describe the dot pattern of a *volume* of uniform intensity.

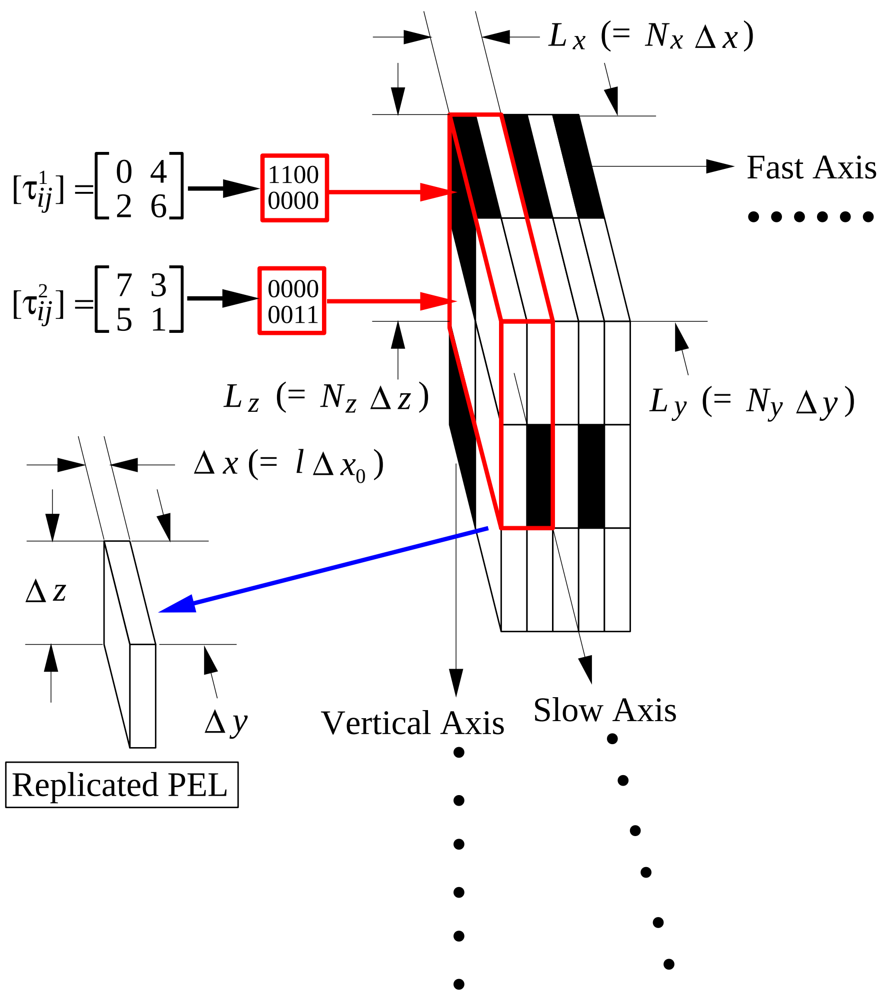

Figure 1: A volume of uniform intensity, level 2
($N_x = N_y = N_z = 2$, $\Delta x_0 = 25$, $\Delta y = 200$, $\Delta z = 175$, $l = 2$)

Generalized Bayer's Algorithm for 3D Halftoning

- **Construction of threshold matrix $[\tau_{ijk}]$**

Suppose that a volume of uniform intensity is represented by repeating a N_x by N_y by N_z subarray of rectangular elements of dimensions Δx, Δy, Δz, respectively – see Figure 1.

Let $I(x, y, z)$ be the intensity value $(= 0$ or $1)$ at $(x, y, z) = (p\Delta x, q\Delta y, r\Delta z)$, where integers $0 \le p < N_x$, $0 \le q < N_y$, $0 \le r < N_z$, then:

$$I(x + N_x\Delta x,\ y,\ z) = I(x,\ y + N_y\Delta y,\ z) = I(x,\ y,\ z + N_z\Delta z) = I(x, y, z), \tag{1}$$

i.e.,

$$\begin{aligned} I(\,(p + N_x)\Delta x,\ q\Delta y\ r\Delta z) &= I(p\Delta x,\ (q + N_y)\Delta y,\ r\Delta z) = \\ I(p\Delta x,\ q\Delta y,\ (r + N_z)\Delta z) &= I(p\Delta x, q\Delta y, r\Delta z). \end{aligned} \tag{1$'$}$$

The basic subarray can be specified by all combinations of such p, q, r and hence,

$$I(x, y, z) = I(p\Delta x, q\Delta y, r\Delta z)$$

$$:= \frac{1}{N_x N_y N_z} \sum_{u=0}^{N_x-1} \sum_{v=0}^{N_y-1} \sum_{w=0}^{N_z-1} J\left(\frac{u}{N_x\Delta x}, \frac{v}{N_y\Delta y}, \frac{w}{N_z\Delta z}\right) e^{i2\pi\left(\frac{up}{N_x} + \frac{vq}{N_y} + \frac{wr}{N_z}\right)}, \tag{2}$$

where,

$$J(\frac{u}{N_x\Delta x}, \frac{v}{N_y\Delta y}, \frac{w}{N_z\Delta z}) := \sum_{u=0}^{N_x-1}\sum_{v=0}^{N_y-1}\sum_{w=0}^{N_z-1} I(p\Delta x, q\Delta y, r\Delta z)e^{-i2\pi(\frac{up}{N_x}+\frac{vq}{N_y}+\frac{wr}{N_z})}. \tag{3}$$

The real part of each term in Eq. (2) is a sinusoidal wave with an *amplitude* A_{uvw}, given by

$$A_{uvw} = \sqrt{J(\frac{u}{N_x\Delta x}, \frac{v}{N_y\Delta y}, \frac{w}{N_z\Delta z})\ J(\frac{-u}{N_x\Delta x}, \frac{-v}{N_y\Delta y}, \frac{-w}{N_z\Delta z})} \tag{4}$$

and a *wave length* λ_{uvw} is given by

$$\lambda_{uvw} = \frac{L_x L_y L_z}{\sqrt{(L_y L_z u)^2 + (L_z L_x v)^2 + (L_x L_y w)^2}} \tag{5}$$

in xyz, where $L_x = N_x\Delta x$, $L_y = N_y\Delta y$, $L_z = N_z\Delta z$, i.e., the length of each edge of a 3D halftone cell.

An *index of texture* Λ in a uniform volume is defined by

$$\Lambda := \max_{u,v,w}\{\lambda_{uvw} | A_{uvw} \neq 0, \lambda_{uvw} < \infty\}, \quad (6)$$

where $\quad 0 \leq u < N_x, \; 0 \leq v < N_y, \; 0 \leq w < N_z.$

$\Longrightarrow$ *Our problem consists of choosing a sequence of* $N_x N_y N_z$ *positions within* N_x *by* N_y *by* N_z *lattice that minimize* Λ *for each level of intensity.*

Example For : $\Delta x_0 = 25\mu$, $\Delta y = 200\mu$, $\Delta z = 175\mu$, $l = 2$

(1) $2 \times 2 \times 2$:

$$\begin{bmatrix} 0 & 4 \\ 2 & 6 \end{bmatrix} \qquad \begin{bmatrix} 7 & 3 \\ 5 & 1 \end{bmatrix}$$

(a) Layer 1 (b) Layer 2

(2) $4 \times 4 \times 4$:

$$\begin{bmatrix} 0 & 34 & 16 & 28 \\ 42 & 58 & 8 & 36 \\ 2 & 32 & 18 & 30 \\ 40 & 56 & 10 & 38 \end{bmatrix} \qquad \begin{bmatrix} 12 & 46 & 24 & 48 \\ 20 & 54 & 4 & 60 \\ 14 & 44 & 26 & 50 \\ 22 & 52 & 6 & 62 \end{bmatrix}$$

(a) Layer 1 (b) Layer 2

$$\begin{bmatrix} 19 & 31 & 3 & 33 \\ 11 & 39 & 41 & 57 \\ 17 & 29 & 1 & 35 \\ 9 & 37 & 43 & 59 \end{bmatrix} \qquad \begin{bmatrix} 27 & 51 & 15 & 45 \\ 7 & 63 & 23 & 53 \\ 25 & 49 & 13 & 47 \\ 5 & 61 & 21 & 55 \end{bmatrix}$$

(c) Layer 3 (d) Layer 4