# Review of Digital Halftoning

**Wonjoon Cho**

**MIT Design Laboratory**

# Outline

- ☐ Introduction
- ☐ Clustered-Dot Ordered Dither
- ☐ Dispersed-Dot Ordered Dither
- ☐ Constrained Average Algorithm
- ☐ Error Diffusion

# Introduction

- Halftone approximation:

Simulation of continuous-tone gray scales for *bilevel* displays and hardcopy devices

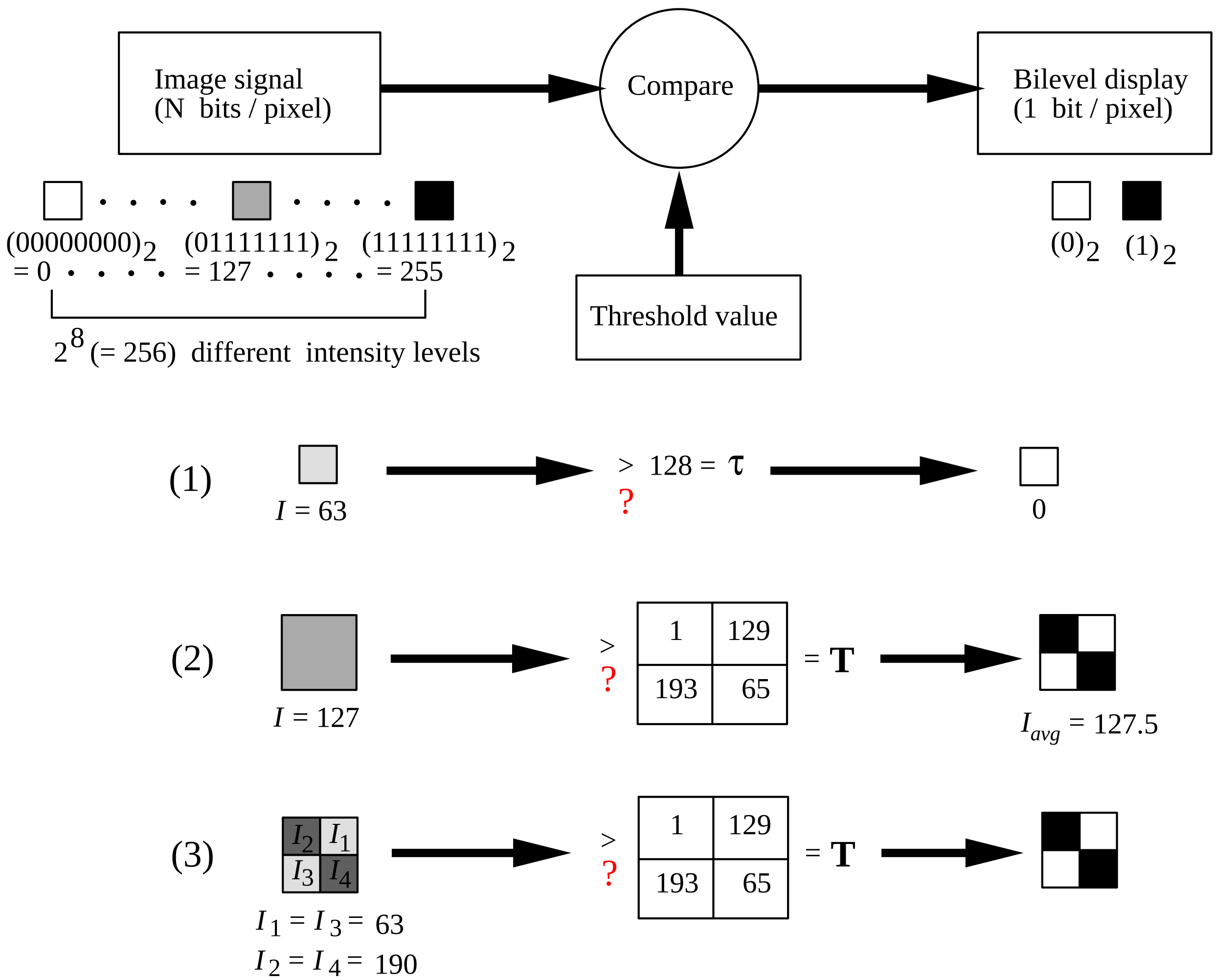



Figure 1: Basic halftoning process

# Introduction (continued)

- Application: Piecewise constant FGM

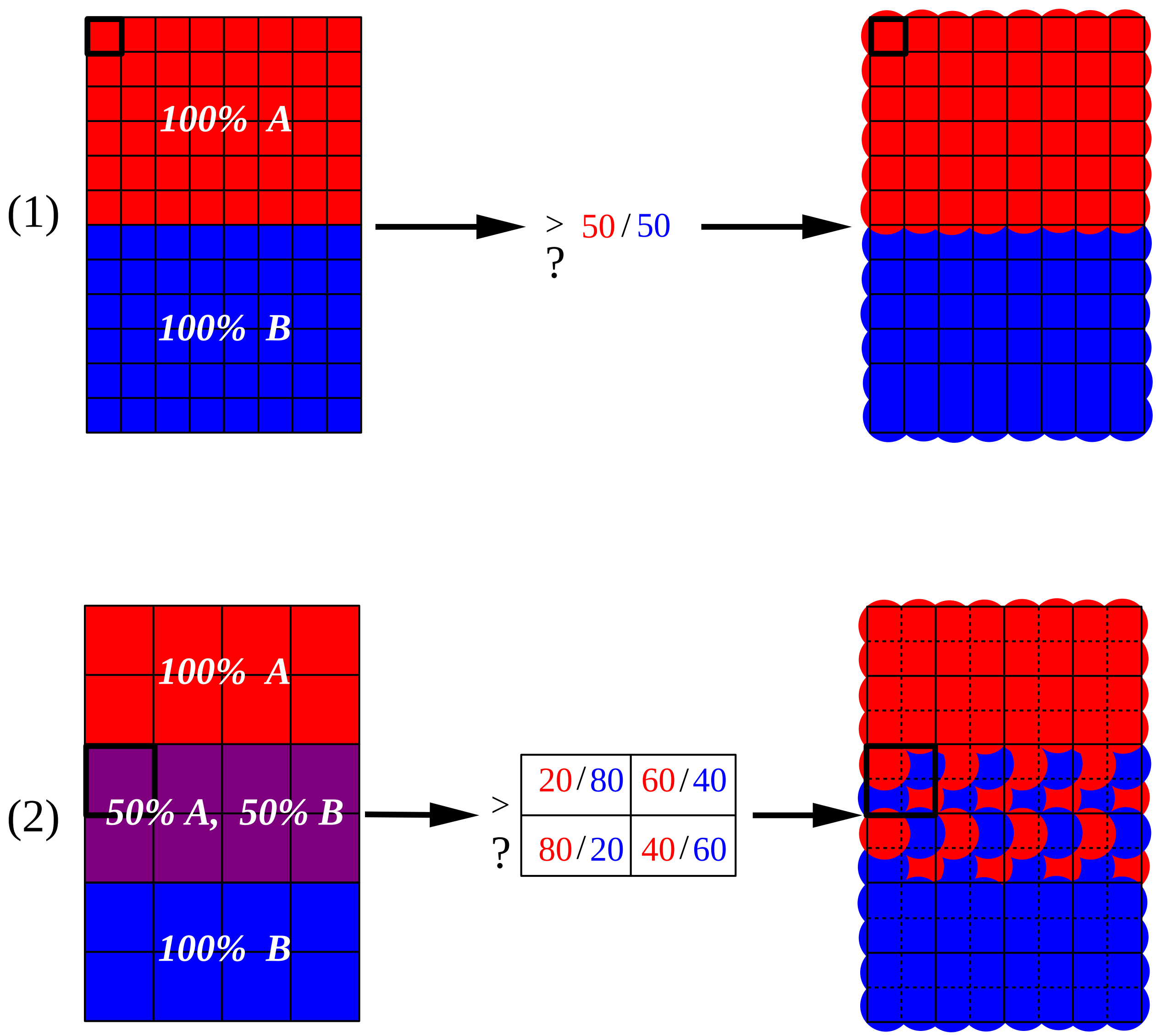
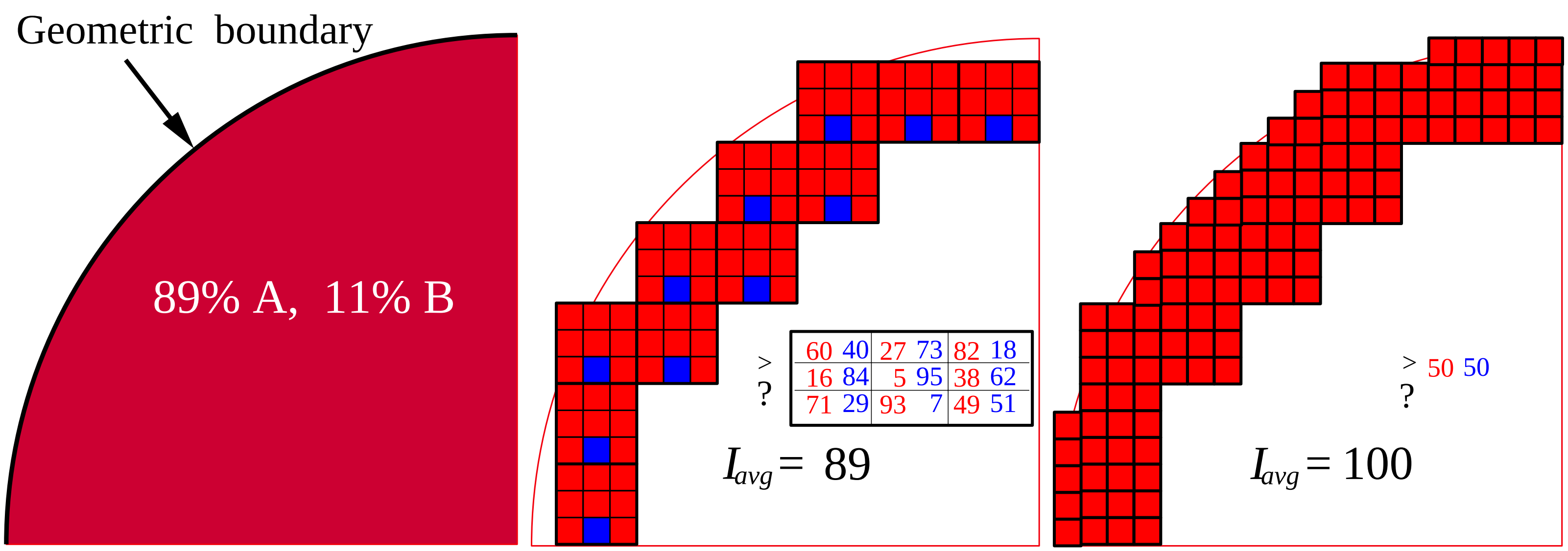



Figure 2: Halftone approximation

# Introduction (continued)

- FGM Example:



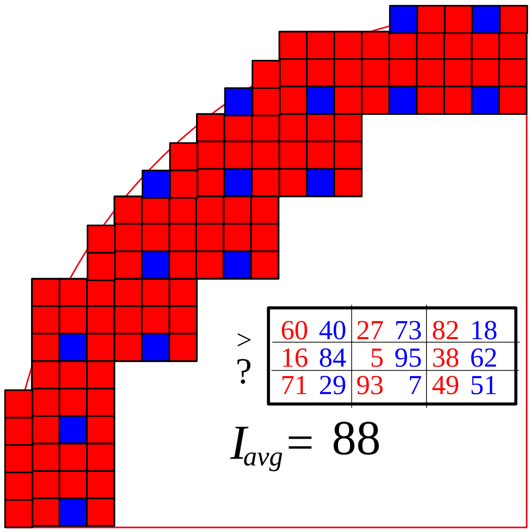

(a) Idealized model (b) 3×3 pixel pattern (c) Fixed threshold

Figure 3: FGM application

## Introduction (continued)

- FGM Example (continued):

Apply $3 \times 3$ pixel pattern with $RES_M = RES_D$, where $RES_M$ and $RES_D$ are the spatial resolution of sampling grid of idealized model and spatial resolution of output device pixel array, respectively.



## Introduction (continued)

- $m \times n$ halftone cell provides $mn + 1$ intensity levels.

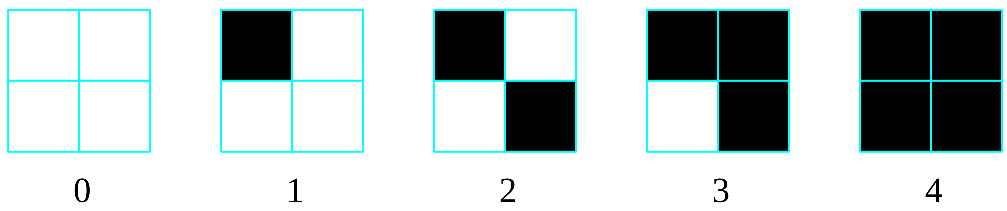



Figure 4: 5 intensity levels $(m = n = 2)$

- Tradeoff: *spatial* resolution / gray scale (*intensity* resolution)

  $\rightarrow$ $8 \times 8$ halftone cell cuts spatial resolution by $\frac{1}{4}$ of $2 \times 2$ halftone cell while providing 13 times more intensity levels.

# Introduction (continued)

- Dither matrix, **T**

To simulate an intensity $I$, we turn on all the elements of $\mathbf{T}$ whose values are less than $I$.

$$\text{Let } \mathbf{T} = \begin{pmatrix} 1 & 129 & 33 & 161 \\ 193 & 65 & 225 & 97 \\ 49 & 177 & 17 & 145 \\ 241 & 113 & 209 & 81 \end{pmatrix}$$

and $I = 127$:

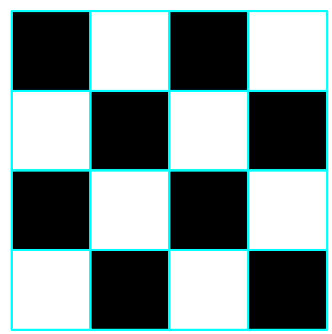


Figure 5: Pixel pattern

# Introduction (continued)

- Fundamental requirements for pixel patterns
  - Minimize visual *artifacts* in the area of *identical* intensity values

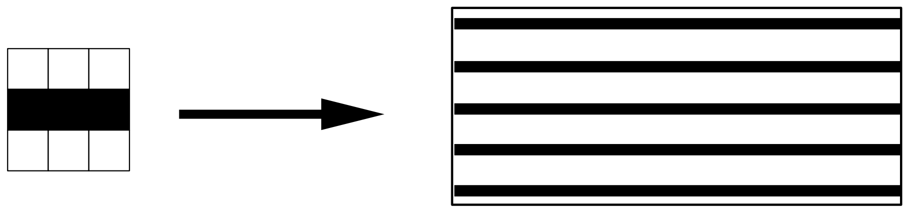


Figure 6: $3 \times 3$ pixel pattern inappropriate for halftoning

  - Growth sequence: Any pixel intensified for intensity level $j$ is also intensified for all levels $k > j \Longrightarrow$ Minimize the *differences* in the patterns for *succesive* intensity levels

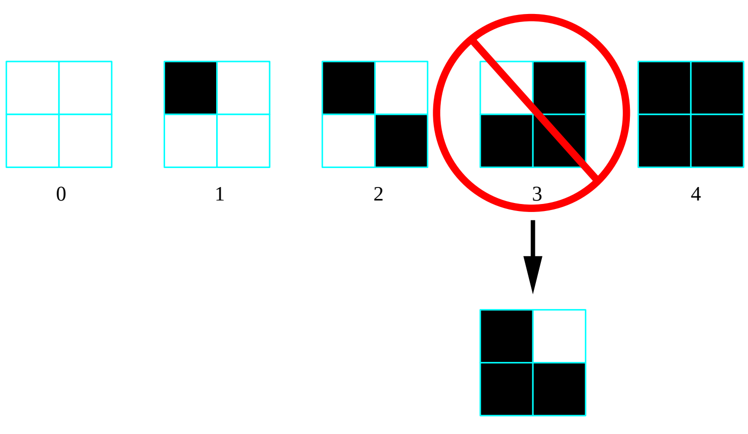



Figure 7: Growth sequence

# Clustered-Dot Ordered Dither

- Ordered dither: Threshold values are *ordered* and not affected by image values.

- Clustered-dot: On-pixels are *adjacent* to each other.

  – If devices are poor at accomodating *isolated* on-pixels, e.g. halftone laser printer, clustered-dot ordered dither is used.

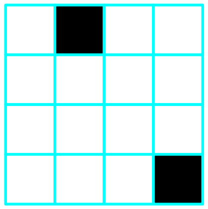


Figure 8: Unacceptable pixel pattern in clustered-dot ordered dither

  – Outward growing pattern $\Longrightarrow$ effect of increasing dot size.

# Clustered-Dot Ordered Dither (continued)

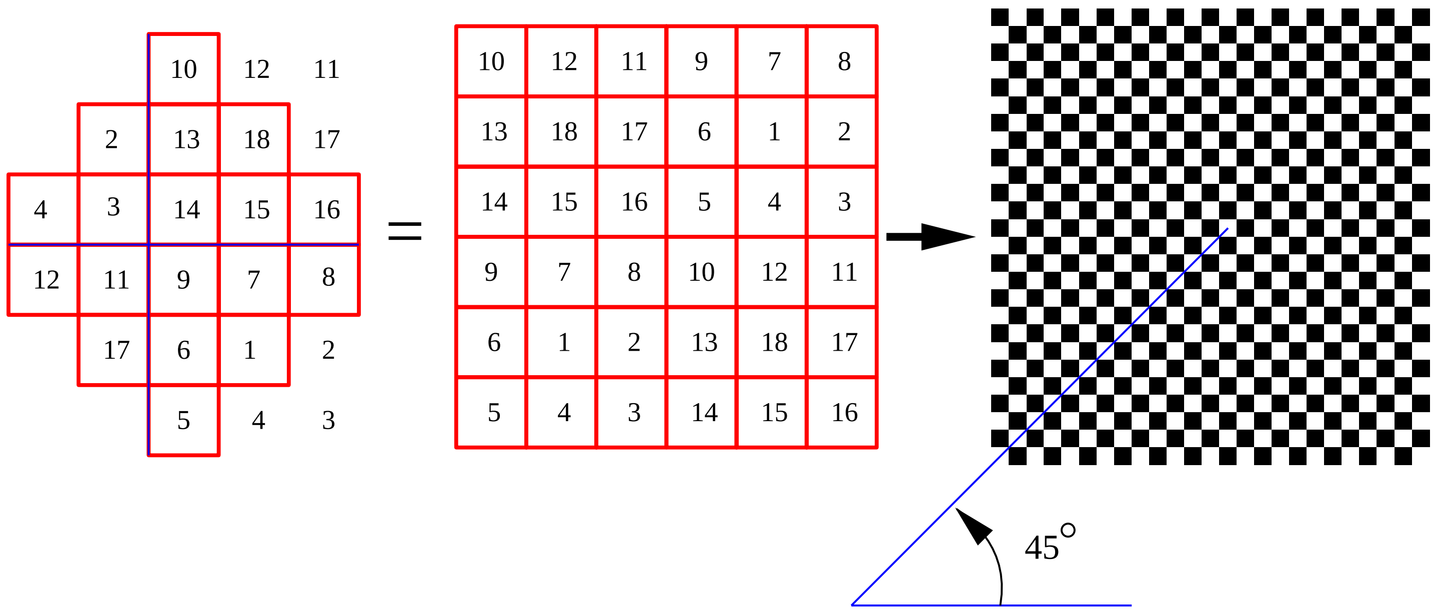



Figure 9: Screen at 45° angle ($\tilde{I} = 0.5$)

## Dispersed-Dot Ordered Dither

- Dispersed-dot: On-pixels are *not* necessarily *adjacent* to each other.
  - Unlike a laser printer, a CRT display is quite able to display *indivisual* dots.

    $\Longrightarrow$ Relax the clustering requirement and dispersed-dot ordered dither can be used.

## Dispersed-Dot Ordered Dither (continued)

B. E. Bayer, An optimum method for two-level rendition of continuous-tone pictures, *International Conference on Communications, Conference Record*, 1973, 11-15.

- **Given:** The *size*, $m$, of a $2^m \times 2^m$ dither matrix.
- **Find:** A rule for *optimum dither pattern* i.e., the optimum *order* in which new dots are added to the halftone cell as gray scale is varied.
- **Optimality:** Minimization of *low frequency texture* in the area of *uniform* gray scale.
  - Visual response to sinusoidal patterns diminishes rapidly as spatial frequency increases.

## Dispersed-Dot Ordered Dither (continued)

**Suppose:** A uniform gray area is represented by repeating (both horizontally & vertically) $2^m \times 2^m$ halftone cell. Then, the normalized intensity $g_{xy}$ ($= 0$ or $1$) at grid point $(x, y)$, $0 \leq x, y < 2^m$, satisfies

$$g_{x+2^m,\, y} = g_{x,\, y+2^m} = g_{xy}$$

Therefore,

$$\Longrightarrow g_{xy} = \sum_{u=-l+1}^{l} \sum_{v=-l+1}^{l} G_{uv} e^{-2\pi i (\frac{ux+vy}{2^m})}$$

$$\Longrightarrow G_{uv} = \frac{1}{2^{2m}} \sum_{x=0}^{2^m-1} \sum_{y=0}^{2^m-1} g_{xy} e^{2\pi i (\frac{ux+vy}{2^m})}$$

where $l = 2^{m-1}$.

$Re(G_{uv} e^{-2\pi i (\frac{ux+vy}{2^m})})$: A sinusoidal plane wave with

- Amplitude: $A_{uv} = \sqrt{G_{uv} G_{-u,-v}}$
- Wave length: $\lambda_{uv} = \frac{2^m}{\sqrt{u^2+v^2}}$

# Dispersed-Dot Ordered Dither (continued)

$$\Lambda = Max\{\lambda_{uv} | A_{uv} \neq 0, \lambda_{uv} < \infty\}$$

where $\lambda_{uv} = \frac{2^m}{\sqrt{u^2+v^2}}$

$\Longrightarrow$ Choose patterns that make $\Lambda$ as small as possible.

- **Rule for optimum dither pattern:**

**(1)** $g_{x\pm 2^{m-1}, y\pm 2^{m-1}} = g_{x\pm 2^{m-1}, y\mp 2^{m-1}} = g_{xy}$

$\longrightarrow$ whenever the number of dots is even.

**(2)** $g_{x\pm 2^{m-1}, y} = g_{x, y\pm 2^{m-1}} = g_{xy}$

$\longrightarrow$ whenever the number of dots is even in the $2^m \times 2^{m-1}$ submatrix.

Example: $4 \times 4$ dither matrix ($m = 2$)

| | | | |
|---|---|---|---|
| 0 | 8 | 2 | 10 |
| 12 | 4 | 14 | 6 |
| 3 | 11 | 1 | 9 |
| 15 | 7 | 13 | 5 |

Figure 10: Dither matrix satisfying Bayer's rule

## Dispersed-Dot Ordered Dither (continued)

- **Example:**

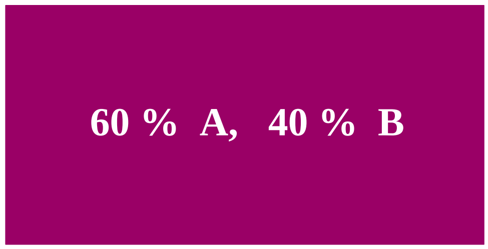



Figure 11: Idealized model

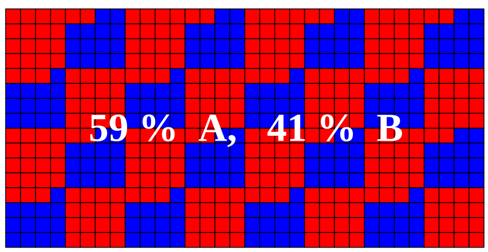



Figure 12: Clustered-dot ordered dither

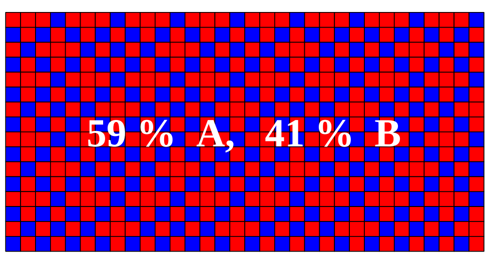



Figure 13: Bayer's dispersed-dot ordered dither

## Dispersed-Dot Ordered Dither (continued)

C. N. Judice and W. H. Ninke, Using ordered dither to display continuous tone pictures on an AC plasma panel, *Proceedings of the Society for Information Display*, 1974, 161-169

- **Given:** A $2 \times 2$ dither matrix $\mathbf{T}^{(2)}$.
- **Find:** A *recurrence relation* to compute $2^m \times 2^m$ dither matrix $\mathbf{T}^{(2m)}$ from $\mathbf{T}^{(m)}$.
- **Optimality:** Minimize the *loss* of spatial resolution when $m$, i.e. intensity resolution, increases.

## Dispersed-Dot Ordered Dither (continued)

$$\mathbf{T}^{(2m)} = \begin{pmatrix} 4\mathbf{T}^{(m)} & 4\mathbf{T}^{(m)} + 2\mathbf{U}^{(m)} \\ 4\mathbf{T}^{(m)} + 3\mathbf{U}^{(m)} & 4\mathbf{T}^{(m)} + \mathbf{U}^{(m)} \end{pmatrix}$$

$$\text{where} \qquad \mathbf{U}^{(m)} = \begin{pmatrix} 1 & 1 & \dots & 1 \\ 1 & 1 & \dots & 1 \\ . & . & \dots & . \\ 1 & 1 & \dots & 1 \end{pmatrix} \text{ and}$$

$$\mathbf{T}^{(2)} = \begin{pmatrix} 0 & 2 \\ 3 & 1 \end{pmatrix}$$

Example: $m = 2$

$$\begin{aligned} \mathbf{T}^{(4)} &= \begin{pmatrix} 4\mathbf{T}^{(2)} & 4\mathbf{T}^{(2)} + 2\mathbf{U}^{(2)} \\ 4\mathbf{T}^{(2)} + 3\mathbf{U}^{(2)} & 4\mathbf{T}^{(2)} + \mathbf{U}^{(2)} \end{pmatrix} \\ &= \begin{pmatrix} 0 & 8 & 2 & 10 \\ 12 & 4 & 14 & 6 \\ 3 & 11 & 1 & 9 \\ 15 & 7 & 13 & 5 \end{pmatrix} \end{aligned}$$

## Dispersed-Dot Ordered Dither (continued)

It is seen that each $2 \times 2$ submatrix *normalized* to image intensity resolution $(2^N)$, where $N$ is # of bits of an image pixel, has the form of:

$$\begin{pmatrix} \alpha & 2 \times 2^{N-2} + \alpha \\ 3 \times 2^{N-2} + \alpha & 1 \times 2^{N-2} + \alpha \end{pmatrix}$$

where $0 \le \alpha < 2^{N-2}$.

Example: $m = 2$ and $N = 8 \Longrightarrow 2^{N-2} = 64$

$$\begin{pmatrix} 0 & 128 \\ 192 & 64 \end{pmatrix}, \begin{pmatrix} 16 & 144 \\ 208 & 80 \end{pmatrix}, \begin{pmatrix} 32 & 160 \\ 224 & 96 \end{pmatrix}, \begin{pmatrix} 48 & 176 \\ 240 & 112 \end{pmatrix}$$

$D^{(2)}$ normalized to $2^N$ is $\begin{pmatrix} 0 & 2 \times 2^{N-2} \\ 3 \times 2^{N-2} & 1 \times 2^{N-2} \end{pmatrix}$

Example: $N = 8 \Longrightarrow \begin{pmatrix} 0 & 128 \\ 192 & 64 \end{pmatrix}$

Therefore, as $m$ increases, the larger dither matrices will still display *intensity gradients* of $D^{(2)}$ over the same distance. In other words, larger dither matrices are built from smaller matrices in a *gradual* way so that ability to reproduce detail or intensity gradients is only weakly dependent on the matrix size.

## Dispersed-Dot Ordered Dither (continued)

- **Example:**

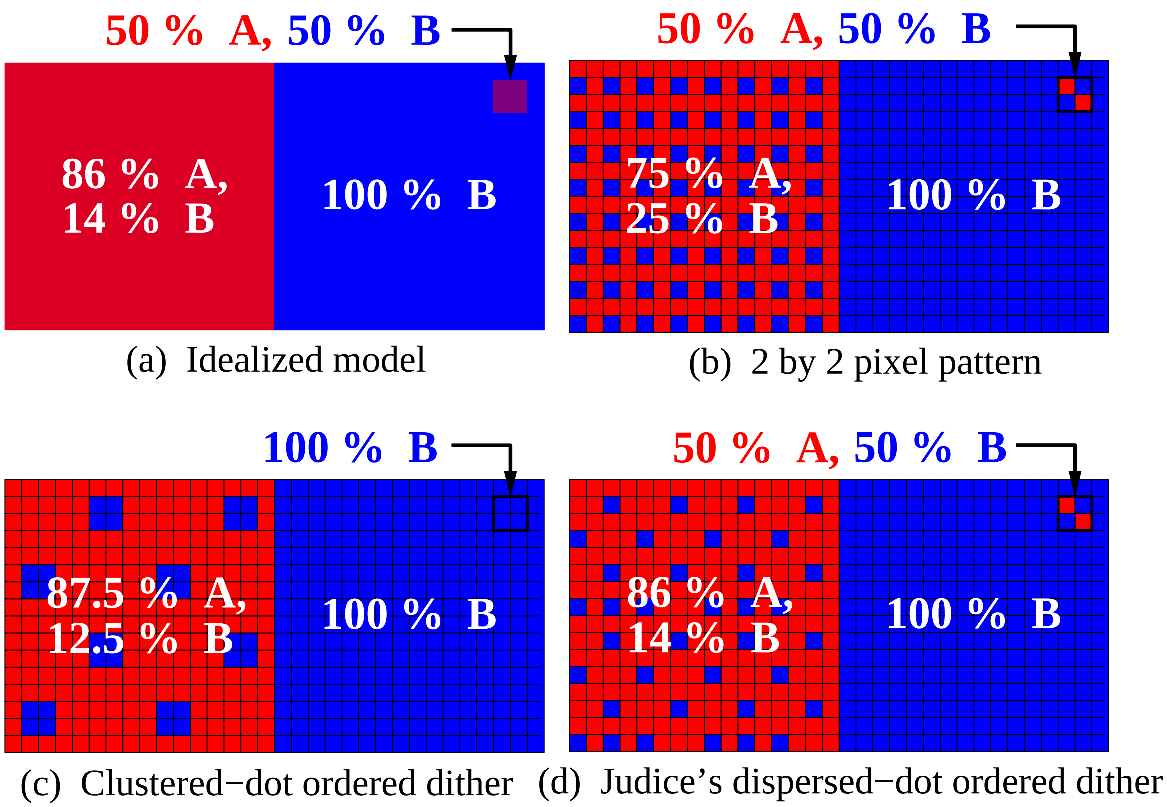
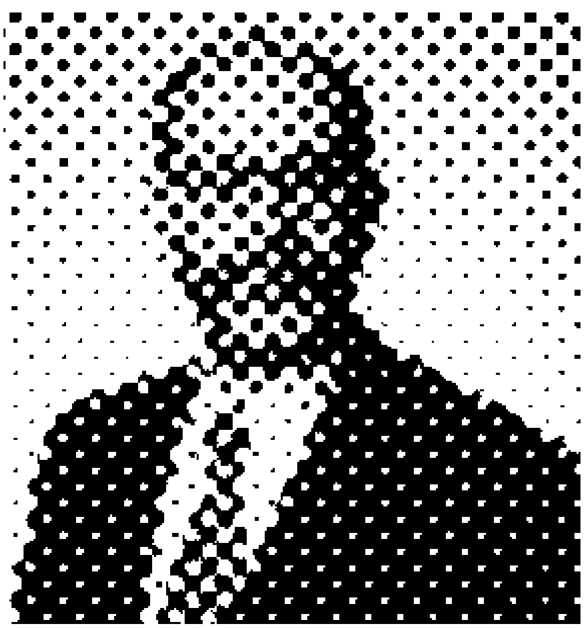



Figure 14: Comparison

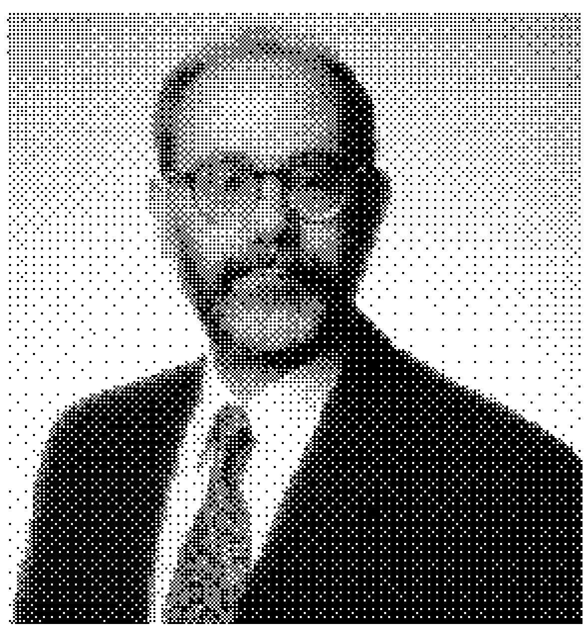
## Examples

(a) Normal gray scale image
(8 bits per pixel)

(b) Simple fixed thresholding
( $\tau = 0.5$ )



(c) Clustered−dot ordered
(129 gray levels)



(d) Dispersed−dot ordered (Bayer's)
(256 gray levels)

Figure: Comparison

## Constrained Average Algorithm

| |
|---|
| J. F. Jarvis and C. S. Roberts, A new technique for the bilevel rendition of continuous tone images on a bilevel display, *IEEE Transactions on Communications*, 24(8), 1976, 891-898 |

- **Given:** An array of image[a] intensities, $I_{xy}$.
- **Find:** *Threshold* value $\tau_{xy}$ for each image pixel.
- **Optimality:** Both *edge emphasis* (increasing legibility of textual information & areas of high detail) and *gray scale rendition* (properly displaying continuous-tone original image).

[a] The input image is assumed to have inherent noise.

## Constrained Average Algorithm (continued)

$$\boxed{\tau_{xy} = \gamma + [1 - \frac{2\gamma}{I_{max}}]\tilde{I}_{xy}}$$

where

- $\tau_{xy}$: Threshold value for an image pixel of intensity $I_{xy}$
- $\gamma$: Parameter conrolling the apparent contrast $(0 \leq \gamma \leq \frac{I_{max}}{2})$ – user input
- $I_{max}$: Maximum image intensity, e.g. for N-bits-per-pixel, $I_{max} = 2^N - 1$
- $\tilde{I}_{xy}$: Average intensity in the neighborhood of an image pixel i.e.,

  $\Longrightarrow \tilde{I}_{xy} = \frac{1}{9}\sum_{i=-1}^{1}\sum_{j=-1}^{1} I_{x+i,y+j}$

$$\boxed{\text{If } I_{xy} > \tau_{xy}, \text{ then } J_{xy} = 1; \text{ else } J_{xy} = 0}$$

where $J_{xy}$ is the intensity of a bilevel output pixel.

## Constrained Average Algorithm (continued)

$$\tau_{xy} = \gamma + [1 - \frac{2\gamma}{I_{max}}]\tilde{I}_{xy}, \qquad (0 \le \gamma \le \frac{I_{max}}{2})$$

- If $\gamma = 0$: $\tau_{xy} = \tilde{I}_{xy}$
  - Maximize the edge emphasis.

    $\Longrightarrow$ Set $\gamma$ small to enhance edge emphasis.

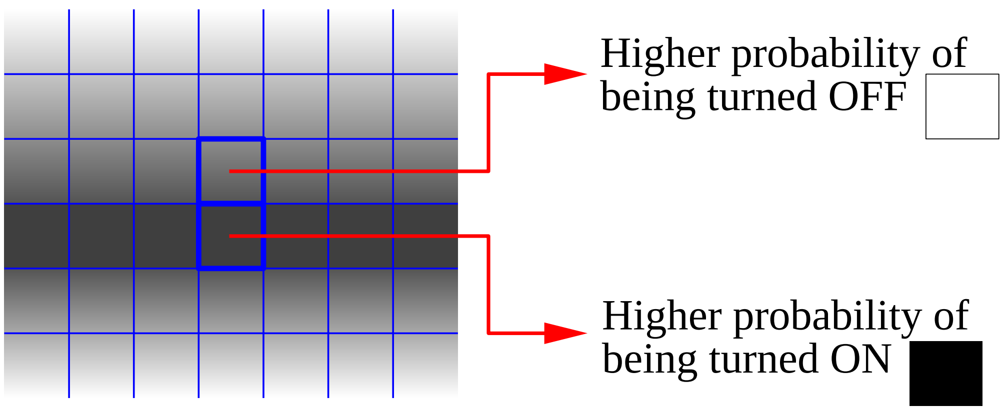



Figure 15: Edge emphasis

- If $\gamma = \frac{I_{max}}{2}$: $\tau_{xy} = \frac{I_{max}}{2}$
  - Simple fixed threshold approach.

    $\Longrightarrow$ No edge emphasis or gray scale rendition.

# Constrained Average Algorithm (continued)

$$\boxed{\tau_{xy} = \gamma + [1 - \frac{2\gamma}{I_{max}}]\tilde{I}_{xy}, \qquad (0 \leq \gamma \leq \frac{I_{max}}{2})}$$

- $\underline{0 \leq \tilde{I}_{xy} \leq I_{max}} \Longrightarrow \boxed{\gamma \leq \tau_{xy} \leq I_{max} - \gamma}$

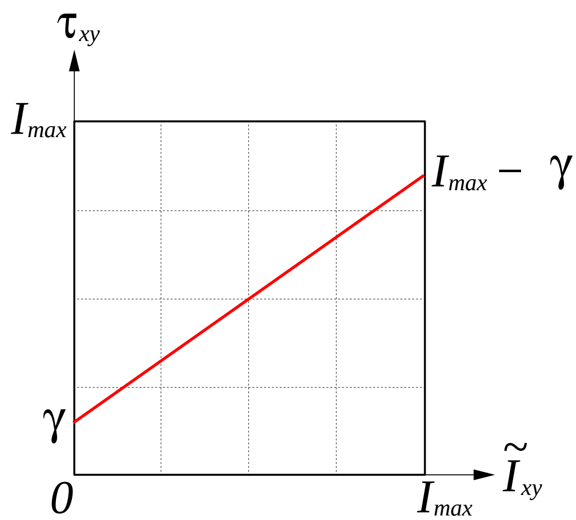



Figure 16: $\tilde{I}_{xy}$ vs. $\tau_{xy}$

- If $\tilde{I}_{xy} > (<) \frac{I_{max}}{2}$, then $\tau_{xy} < (>) \tilde{I}_{xy} \Longrightarrow$ more (less) than $\frac{1}{2}$ of the output pixels will be turned ON, respectively.
- Choosing $\gamma \sim \frac{L_N}{2}$ removes *noisy dots* in the area of $\tilde{I}_{xy} \sim 0$ or $\tilde{I}_{xy} \sim I_{max} \Longrightarrow$ As $\tilde{I}_{xy}$ approaches 0 ($I_{max}$), nearly all the output pixels will be turned OFF (ON), respectively.

  $*$ $L_N$: width of noise distribution.

## Error Diffusion

R. Floyd and L. Steinberg, An adaptive algorithm for spatial gray scale, *Society for Information Display 1975 Symphosium Digest of Technical Papers*, 1975, 36–37

- **Given:** An array of image intensities, $I_{xy}$.
- **Spread:** $error = I_{xy} - J_{xy}$ over neighboring pixels.
- **Optimality:** Minimize global error in halftoning process.

## Error Diffusion (continued)

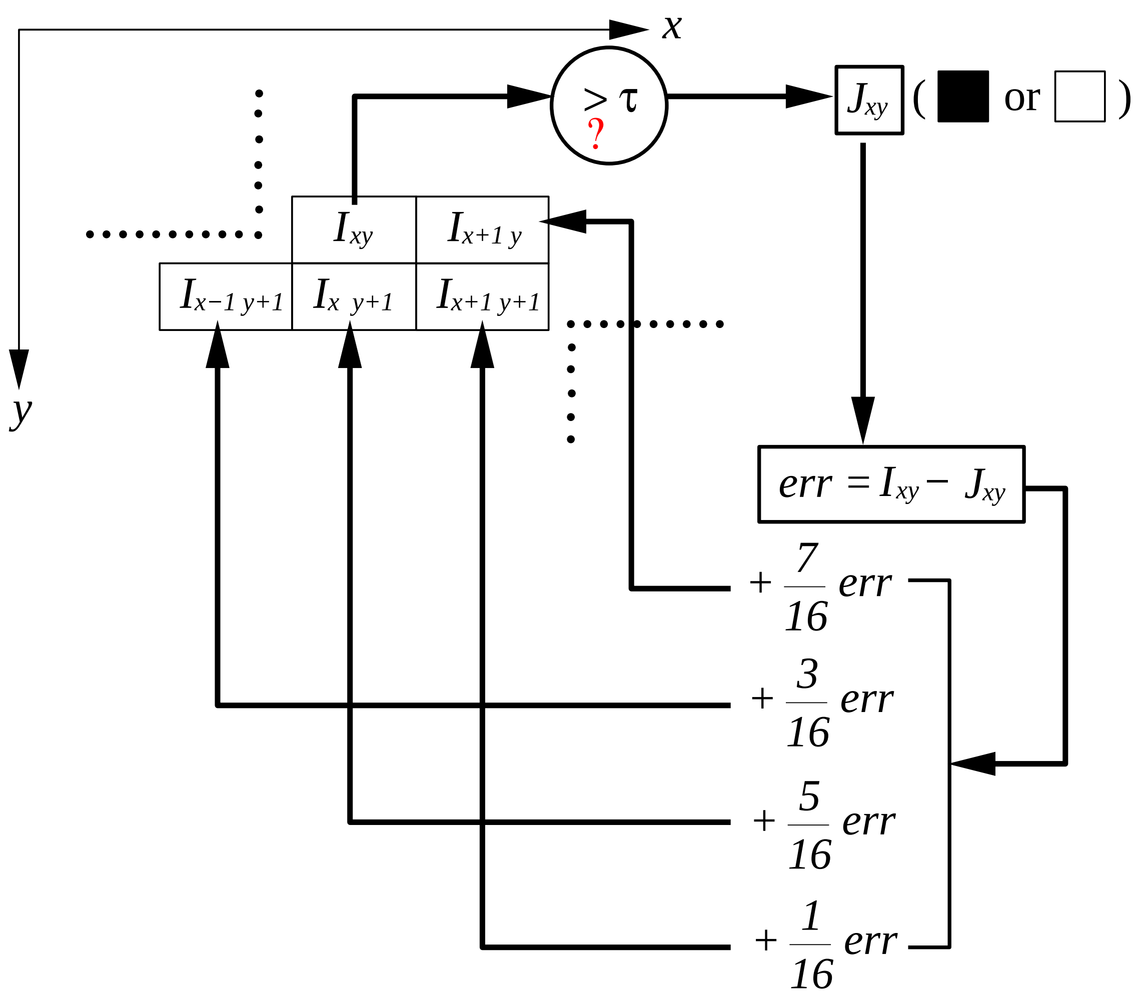



Figure 17: Error diffusion process

# Error Diffusion (continued)

(a) Normal gray scale image (b) Error diffusion (Floyd & Steinberg)

Figure 18: Example

- Gray levels are represented by pleasingly structureless distribution of dots.
- Transient behavior near abrupt edges or boundaries.